Reg. No. : | | | | | | | | | | |

Question Paper Code: 45021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

Fifth Semester

Computer Science and Engineering

14UMA521 - DISCRETE MATHEMATICS

(Regulation 2014)

(Common to IT Branch)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Let $P(x): x < 32$ and $Q(x): x$ is a multiple of 10 with universe of discourse as all positive integers. Then the truth value of $(\exists x)(P(x) \rightarrow Q(x))$is
   (a) True (b) False (c) 10 (d) 20

2. $P \rightarrow Q$is equivalent to
   (a) $\urcorner Q \rightarrow P$ (b) $Q \rightarrow P$ (c) $P \rightarrow \urcorner Q$ (d) $\urcorner Q \rightarrow \urcorner P$

3. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?
   (a) 620 (b) 710 (c) 720 (d) 610

4. The numbers of ways in which 6 boys and 4 girls be arranged in a straight line so that no two girls are together is
   (a) 10^{P_6} (b) 604800 (c) 720 (d) 17280

5. A vertex of degree one is called
   (a) Isolated vertex (b) Unit vertex (c) Pendant vertex (d) Proper vertex

6. The number of vertices in a regular graph of degree 4 with 10 edges is

(a) 4 (b) 10 (c) 6 (d) 5

7. A ring with identity $(R, +, .)$ is a field if

(a) $(R, +)$ is commutative

(b) Every non-zero element has a multiplicative inverse

(c) Both (a) and (b)

(d) Only (b) not (c)

8. The necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup when $a, b \in H$is

(a) $a^{-1} * h * a \in H$ (b) $a^{-1} * b \in H$

(c) $a^{-1} * b^{-1} \in H$ (d) $(a * b)^{-1} \in H$

9. The value of $(a.b)' + (a+b)'$ is

(a) $a'.b'$ (b) $a' + b'$ (c) 0 (d) 1

10. The value of $(\mathrm{a.b})' + (\mathrm{a+b})'$ is

(a) $a'.b'$ (b) $a' + b'$ (c) 0 (d) 1

PART - B (5 x 2 = 10 Marks)

11. Using truth table, show that $P \vee \urcorner (P \wedge Q)$ is tautology.

12. Find the recurrence relation from $y_k = A2^k + B3^k$.

13. Give an example of a graph which is both Eulerian and Hamiltonian.

14. Draw all the spanning trees of $\boldsymbol{K_3}$.

15. Let A={ a, b, c } and $\rho(A)$ be its power set. Draw the Hasse diagram of $(\rho(A), \subseteq)$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Obtain the principal disjunctive and principal conjunctive normal forms of $\big(P \to (Q \wedge R)\big) \wedge \big(\sim P \to (\sim Q \wedge \sim R)\big)$. (8)

(ii) Show that $T \wedge S$ can be derived from the premises $P \to Q, Q \to \sim R, R, P \vee (T \wedge S)$. (8)

Or

(b) (i) Check whether the following set of premises are not valid: Whenever the system software is being upgraded, users cannot access the file system. If users can access the file systems, then they can save new files. If users cannot save new files, then the system software is not being upgraded. (8)

(ii) Use the indirect method to prove that the conclusion $(\exists z)Q(z)$ follows from the premises$(\forall x)(P(x) \rightarrow Q(x))$and $(\exists y)P(y)$. (8)

17. (a) (i) Solve the recurrence relation $y_{n+2} - 6y_{n+1} + 9y_n = 0,\ y_1 = 4\ and\ y_0 = 1.$ (8)

(ii) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$, given that $a_0 = 2$ and $a_1 = 8$. (8)

Or

(b) (i) Show that by mathematical induction principle, $3^{2n} + 4^{n+1}$ is divisible by 5, for $n \geq 0$. (8)

(ii) Find the number of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 and 7. (8)

18. (a) (i) Find the adjacency matrix of the following graph G.

Find $A^2, A^3$and $Y = A + A^2 + A^3 + A^4$. What is your observation of entries in A^2andA^3? (8)

(ii) Prove that a connected graph is Eulerian if all vertices are of even degree. (8)

Or

(b) (i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (8)

(ii) Find the adjacency matrix of the following graph G. Find $A^2, A^3$and $Y = A + A^2 + A^3 + A^4$. What is your observation of entries in $A^2$and A^3 ? (8)

19. (a) (i) Let $*$ be defined on R by $x * y = x + y + 2xy\ \forall x, y \in R$. Check whether $(R,*)$ is a monoid (or) not. Is it commutative? Also find the inverses of $(R,*)$. (8)

(ii) State and prove Lagrange's theorem. (8)

Or

(b) (i) Let G be a group. If $a, b \in G$, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ (8)

(ii) Define subgroup with an example. Also prove that the intersection of two subgroups of a group is also a subgroup of the group. (8)

20. (a) (i) Prove that De Morgan's laws hold good for a complemented distributive lattice. (8)

(ii) In a Boolean algebra, prove that the following statements are equivalent: (i) $a + b = b$ (ii) $a.b = a$ (iii) $a' + b = 1$ (iv) $a.b' = 0$. (8)

Or

(b) (i) In a complemented, distributive lattice, prove the following:

(i) $(ab)' + (a + b)' = a' + b'$ (ii) $ab'c + ab'c = b'c$ (8)