Reg. No. : | | | | | | | | | | |

Question Paper Code: 44024

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

Fourth Semester

Electronics and Communication Engineering

14UMA424 - PROBABILITY AND RANDOM PROCESS

(Regulation 2014)

Duration: Three hours　　　　　　　　　　　　　　　Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. The probability of impossible event is
   (a) 1　　(b) 0　　(c) 2　　(d) 0.5

2. In which probability distribution, Variance and Mean are equal?
   (a) Binomial　　(b) Poisson　　(c) Geometric　　(d) None of these

3. If two random variables X and Y are independent, then covariance is
   (a) θ　　(b) 1　　(c) 0　　(d) λ

4. If $X=Y$ then correlation coefficient between them is
   (a) 0　　(b) ∞　　(c) 1　　(d) ± 1

5. Every Strongly stationary process of order 2 is a
   (a) Orthogonal process　　(b) Stationary Process
   (c) WSS Process　　(d) None of these

6. If both T and S are discrete, then the random process is called
   (a) stationary　　(b) discrete random sequence
   (c) random process　　(d) Poisson process

7. Autocorrelation function is an ____________ function.
   (a) odd　　(b) even
   (c) neither Even nor odd　　(d) stationary

8. If $R_{xy}(\tau) = \mu_X \times \mu_Y$ then *X(t)* and *Y(t)* are called

(a) Independent (b) Orthogonal

(c) Stationary (d) none of these

9. Which of the following system is Causal?

(a) *y(t)=x(t+a)* (b) *y(t)= x(t-a)*

(c) *(t)= a x(t+a)* (d) *y(t)= x(t) – x(t-a)*

10. Colouted Noise means a noise that is

(a) white (b) not white

(c) coloured (d) none of these

PART - B (5 x 2 = 10 Marks)

11. If a Random variable *X* has the moment generating function $M_x(t)=\frac{2}{2-t}$. Determine the variance of *X*.

12. Let *X* and *Y* be random variables with joint density function *f(x, y)=2-x-y* in $0\leq x<y\leq 1$, fomulate the value of *E(x)*?

13. Outline discrete random process. Give an example for it.

14. Devise the properties of auto correlation function.

15. If *X(t)* is a WSS process and if $Y(t)= \int_{-\infty}^{\infty} h(u)X(t-u)du$, then formulate $R_{XY}(\tau) = R_{XY}(\tau) * h(\tau)$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) If the probability density function of a random variable *X* is given by $f(x) = K x^2 e^{-x}, x \geq 0$. Identify the value of *K*, r^{th} moment, Mean and Variance. (8)

(ii) Establish the memory less property of Geometric Distribution. (8)

Or

(b) A random variable *X* has the following probability function

Value of *x*	0	1	2	3	4
P (*x*)	*k*	3*k*	5*k*	7*k*	9*k*

Find the value of *k*, *P* (*x* < 3) and distribution function of *x*. (16)

17. (a) (i) If the joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1$.

Find out (i) P(X > 1) , (ii) P(Y< ½). (8)

(ii) Analyse the correlation coefficient between the heights (in inches) of fathers X and their sons Y from the following data. (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

Or

(b) (i) The two lines of regression are $8x - 10y + 66 = 0, 40x - 18y - 214 = 0$. The variance of x is 9. Evaluate the mean values of x and y and the Correlation coefficient between x and y. (8)

(ii) If X and Y are independent variants uniformly distributed in (0, 1). Identify the distribution of XY. (8)

18. (a) (i) Explain the classification of random process. (8)

(ii) The transition probability of a Markov chain $\{X\}$, n = 1, 2, 3……., having 3 states 1, 2 and 3 $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (1) $P\{X_2 = 3\}$ and (2) $P = \{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$. (8)

Or

(b) Generalize the postulates of a Poisson process and derive the probability distribution for the Poisson Process. Also Show that the sum of two independent Poisson process is again a Poisson process. (16)

19. (a) (i) Define cross-correlation function and write the properties of cross-correlation function. (8)

(ii) State and Prove Wiener-Khinchine theorem. (8)

Or

(b) (i) State and Prove Wiener-Khinchine theorem. (8)

(ii) Given that the autocorrelation function of a stationary random process is $R_{xx}(\tau) = \frac{25\tau^2+36}{6.25\tau^2+4}$. Predict the mean and variance of the process $\{X(t)\}$. (8)

20. (a) (i) Show that $S_{yy}(\omega) = S_{xx}(\omega)|H(\omega)|^2$ where Sxx(ω) and Syy(ω) are the power spectral density functions of the input *X(t)*, output *Y(t)* and *H(ω)* is the system transfer function. (8)

(ii) If the input to a time- invariant, stable linear system is a WSS process, Enumerate that the output will also be a WSS process. (8)

Or

(b) (i) A system has an impulse response h(t)=$e^{-\beta t}\ U(t)$, predict the power spectral density of the output Y(t) corresponding to the input X(t). (8)

(ii) If *X(t)* is a band limited process such that $S_{xx}(\omega) = 0, |\omega| > \sigma$, then formulate $2[R_{xx}(0) - R_{xx}(\tau)] \leq \sigma^2\tau^2R_{xx}(0)$. (8)