Reg. No. : | | | | | | | | | | |

Question Paper Code: 44021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

Fourth Semester

Computer Science and Engineering

14UMA421 - APPLIED STATISTICS AND QUEUEING NETWORKS

(Common to Information Technology)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. The cdf of a random variable X is $F(x) = 1 - (1 + x)e^{-x}, x > 0$. The pdf of X is
   (a) x (b) xe^{x} (c) xe^{-x} (d) x^2

2. For the following density function $f(x) = ae^{-|x|}, -\infty < x < \infty$. The value of 'a' is
   (a) 1/2 (b) 0 (c) 1 (d) 2

3. If X and Y are independent , then f(x, y) =
   (a) f(x) (b) f(x)f(y) (c) f(y) (d) f(x) + f(y)

4. If $X_1, X_2, \ldots, X_n, \ldots$ is a sequence of independent RVs with $E(X_i) = \mu_i$ and $Var(X_i) = \sigma_i^2, i = 1,2, \ldots$ and if $S_n = X_1 + X_2 + \cdots + X_n$, then under certain general conditions $S_n$follows a
   (a) Binomial distribution (b) Poisson
   (c) Normal (d) Exponential

5. Latin square design are most widely used in the field of
   (a) industry (b) medicine (c) agriculture (d) astronomy

6. The RBD is available for a wide range of treatments
   (a) 1 to 12 (b) 2 to 24 (c) 2 to 29 (d) 1 to 29

7. The process in which customer jumps from one queue to another to get service
   (a) Balking (b) Reneging (c) Priority (d) Jockeying

8. The effective arrival rate λ' is given by
   (a) $\mu(1-\rho)$ (b) $\mu(1-P_0)$ (c) $\mu(1+\rho)$ (d) $\mu(1+P_0)$

9. If there are 2 servers in an infinite capacity Poisson queue system with $\lambda = 10$ per hour and $\mu = 15$ per hour, what is the percentage of idle time for each server?
   (a) 33.33% (b) 66.66% (c) 25% (d) 75%

10. No customer may enter the system from outside
   (a) Jackson (b) Open Jackson
   (c) Closed Jackson (d) None of these

PART - B (5 x 2 = 10 Marks)

11. A continuous RV has a pdf $f(x) = kx^2e^{-x}, x \geq 0$. Find k, mean and variance.

12. The joint probability mass function of (X, y) is given by $p(x, y) = k)2x + 3y), x = 0,1,2; y = 1,2,3$. Find the marginal distribution and also find the joint probability distribution of X + Y.

13. What are the basic principles of the design of experiments?

14. Define Kendal's Notation.

15. Define Open Jackson Networks?

PART - C (5 x 16 = 80 Marks)

16. (a) (i) A bag contains 5 balls and it is not know how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the change that all the balls in the bag are white? (8)

   (ii) Find the moment generating function of an exponential distribution and hence find the mean and variance. (8)

Or

   (b) (i) A coin is tossed until a head appears or until it has been tossed three times. Given that the head does not appear on the first toss, find the probability that the coin is tossed three times. (8)

(ii) Suppose the height of men of a certain country are normally distributed with average 68 and standard deviation 2.5, find the percentage of country men who are (i) between 66 and 71 (ii) Approximately 6 fit tall. (8)

17. (a) Two random variables X and Y have joint probability density function $f(x,y)=\begin{cases} c(4-x-y), 0\le x\le 2, 0\le y\le 2 \\ 0, \quad \textit{elsewhere} \end{cases}$. Find the equation of two lines of regression.

(16)

Or

(b) (i) The joint probability function (X,Y) is given by $P(x,y)=K(2x+3y), x=0,1,2;\ \ y=1,2,3.$ Find the marginal distributions and P(X + Y). (8)

(ii) Following table gives the data on rainfall and discharge in a certain river. Obtain the Line of regression of Y on X. (8)

Rainfall (X)	1.53	1.78	2.60	2.95	3.42
Discharge (Y)	33.5	36.3	40	45.8	53.5

18. (a) The following data represent the number of units of production per day turned out by different workers using 4 different types of machines.

		Machine Type			
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(i) Test whether the five mean differ with respect to mean productivity

(ii) Test whether the mean productivity is the same for the four different machine types. ($F_{0.05}(4,12)=3.26\,; F_{0.05}(3,12)=3.49$). (16)

Or

(b) Analyze the variance in the Latin square of yields (in kgs) of paddy where P,Q, R, S denote the different methods of cultivation.

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields. ($F_{0.05}\,(3,6) = 4.76$). (16)

19. (a) Arrivals of a telephone in a both are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assured to be distributed exponentially with mean 4 minutes.

(1) Find the average number of persons waiting in the system?

(2) What is the probability that a person arriving at the booth will have to wait in the queue?

(3) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

(4) Estimate the fraction of the day when the phone will be in use.

(5) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 minutes for phone. By how much the flow of arrivals should increase to justify a second booth?

(6) What is the average length of the queue that forms from time to time? (16)

Or

(b) (i) In a railway marshalling yard, goods trains arrive at the rate of 30 trains per day. Assume that the inter-arrival time follows as exponential distribution and the service time is also to be assumed as exponential with mean of 36 minutes. Calculate (1) the probability that the yard is empty (2) the average queue length, assuming the line capacity of the year is 9 trains. (8)

(ii) A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Find the average number of cars in the service station, and the average number of cars waiting for service. (8)

20. (a) Derive Pollaczek – Khinchine Formula. (16)

Or

(b) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for all cars is constant and equal to 10 minutes, determine Ls, Lq, Ws, Wq. (16)