Reg. No. : | | | | | | | | | | |

Question Paper Code: 43021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The formula for finding the Euler constant a_n of a Fourier series in *[0, 2π]* is ________

   (a) $a_n = \int_0^{\pi} f(x) cosnx\, dx$ (b) $a_n = \int_0^{2\pi} f(x) cosnx\, dx$

   (c) $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) cosnx\, dx$ (d) $a_n = \frac{1}{\pi}\int_0^{\pi} f(x) cos\frac{n\pi x}{l}\, dx$

2. If *f(x)* is an odd function defined in $(-l\ ,\ l)$ the value of a_n is _________

   (a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2n}{\pi}$ (d) 1

3. The Fourier cosine transform of e^{-3x} is _________

   (a) $\frac{2}{\pi}\left(\frac{3}{s^2+3^2}\right)$ (b) $\left(\frac{3}{s^2+3^2}\right)$ (c) $\sqrt{\frac{2}{\pi}}\left(\frac{3}{s^2+3^2}\right)$ (d) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+3^2}\right)$

4. If Fourier Transform of *f(x) = F(s)* then the Fourier Transform of *f(ax)* is ______

   (a) $\frac{1}{s} F\left(\frac{s}{a}\right)$ (b) $F\left(\frac{s}{a}\right)$ (c) $\frac{1}{2} F\left(\frac{s}{a}\right)$ (d) $\frac{1}{|a|} F\left(\frac{s}{a}\right)$

5. $\lim_{z \to 1}$ (z-1) F(z) =

   (a) f(1) (b) $F(\infty)$ (c) $f(\infty)$ (d) f(0)

6. $Z\{cosn\theta\}$ is ________

(a) $\frac{(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (b) $\frac{Z\,(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (c) $\frac{Z}{Z^2-2Z\,cos\theta+1}$ (d) $\frac{1}{Z^2-2Z\,cos\theta+1}$

7. When the ends of a rod is non zero for one dimensional heat flow equation, the temperature function *u(x,t)* is modified as the sum of steady state and transient state temperatures. The transient part of the solution which,

(a) increases with increase of time (b) decreases with increase of time
(c) increases with decrease of time (d) increases with decrease of time

8. Classify the partial differential equation $4u_{xx} = u_t$.

(a) Hyperbolic (b) parabolic (c) Elliptic (d) Poisson

9. The finite difference approximation to $y_i' \;=$

(a) $\frac{y_{i+1} - y_{i-1}}{h}$ (b) $\frac{y_{i+1} + y_{i-1}}{h}$
(c) $\frac{y_{i+1} - y_{i-1}}{2h}$ (d) $\frac{y_{i+1} + y_{i-1}}{2h}$

10. The Laplace equation is

(a) $\nabla^2 u = \nabla x$ (b) $\nabla^2 u = f(x, y)$
(c) $\nabla^2 u - f(x, y) = 0$ (d) $\nabla^2 u = 0$

PART - B (5 x 2 = 10 Marks)

11. Find the Fourier constants b_n for $x\, sinx$ in $(-\pi, \pi)$.

12. Find the Fourier sine transform of $\frac{1}{x}$.

13. Find the Z transform of $x(n) = \begin{cases} \frac{a^n}{n!} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

14. Classify $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8\,u_y - 16u = 0.$

15. Derive the explicit difference equation corresponding to the partial differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Express $f(x) = (\pi - x)^2$ as a Fourier series of periodicity 2π *in* $0 < x < 2\pi$. (8)

(ii) Find the Fourier series for $f(x) = x^2$ *in* $-\pi, \ x < \pi$. (8)

Or

(b) (i) Find the cosine series for f (x) = x in (0, π) and then using Parseval's theorem, show that $\frac{1}{1^4} + \frac{1}{3^4} + ... = \frac{\pi^4}{96}$. (8)

(ii) Find the complex form of Fourier series of $f(x)$ if $f(x) = \sin ax \ in \ -\pi < x < \pi$. (8)

17. (a) Find the Fourier Transform of $f(x) = f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

Hence show that (1) $\int_0^\infty \frac{sins - s\ coss}{s^3} \ cos\frac{s}{2} \ ds = \frac{3\pi}{16}$

(2) $\int_0^\infty \frac{(xcosx - sinx)^2}{x^6} \ dx = \frac{\pi}{15}$ (16)

Or

(b) (i) Show that Fourier Transform of $f(x) = e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(4 + x^2)\ (25 + x^2)}$ using Fourier transform method. (8)

18. (a) (i) Find Z $(t^2 e^{-t})$ and Z $(sin^3 \frac{n\pi}{6})$ (8)

(ii) Find Z $^{-1}\left(\frac{z^2 - 3z}{(z-5)(z+2)}\right)$ using residue theorem. (8)

Or

(b) (i) Solve by Z-transforms $y(n+2) + 6y(n+1) + 9y(n) = 2^n$ given $y(0) = y(1) = 0$. (8)

(ii) Using convolution theorem, find the inverse Z-Transform of $\frac{z^2}{(z-1)(z-3)}$. (8)

19. (a) A tightly stretched string with fixed end points $x = 0$ and $x = 10$ is initially in a position Given by $y(x, 0) = k(10x - x^2)$. It is released from rest from this position. Find the expression for the displacement at any time 't'. (16)

Or

(b) A metal bar 20cm long, with insulated sides, has its ends A and B kept at 30ºC and 90ºC respectively until steady state conditions prevail. The temperature at each end is suddenly raised to 0ºC and kept so. Find the subsequent temperature at any time of the bar at any time. (16)

20. (a) Solve the Poisson's equations $\nabla^2 u = -81xy$, $0 < x < 1$, $0 < y < 1$, h=1/3, $u(0,y) = u(x,0)$, $u(1,y) = u(x,1) = 100$. (16)

Or

(b) (i) Solve u_{xx} =32 u_t with h=0.25 for t>0; 0<x<1 and u(x,0)=u(0,t)=0; u(1,t)=t .Tabulate u upto t=5 sec using Bender-Schmidt formula. (8)

(ii) Find the solution to the wave equations $u_{xx} = u_{tt}$,0 < x < 1, t > 0, given that $u_t(x,0)=0$, $u(1,t) = u(0,t) = 0$ and $u(x,0) = 100 \sin \pi x$. Compute u for 4 time steps with h=0.25. (8)
