Reg. No. : | | | | | | | | | | |

Question Paper Code: 34024

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours   Maximum: 100 Marks

(Statistical tables may be permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Find *c*, if a continuous random variable *X* has the density function $f(x) = \dfrac{c}{1+x^2}, -\infty \le x \le \infty$.

2. The moment generating function of a random variable *X* is given by $(t) = e^{2(e^t-1)}$. What is $P[X=0]$?

3. The regression equations are $3X+2Y=26$ and $6X+Y=31$. Find the correlation coefficient between *X* and *Y*.

4. The joint p.d.f of the *RV* *(X,Y)* is given by $f(x,y) = kxye^{-(x^2+y^2)}, x>0, y>0$. Find the value of *k*.

5. Define a Markov chain and give an example.

6. Consider a Markov chain with state {0, 1} transition probability matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Is the state 0 periodic? If so, what is the period?

7. State any two properties of cross correlation function.

8. Show that the spectral density function of a real random process is an even function.

9. Define a system. When it is called linear system?

10. Prove that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$3k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) the value of k, (ii) $P(1.5 < X < 4.5 \,/\, X > 2)$ and (iii) the smallest value of λ for which $P(X \leq \lambda) > \frac{1}{2}$. (8)

(ii) Show that sum of any two Poisson variables is again a Poisson variate. (8)

Or

(b) (i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act independently.

(1) What is the probability that all four lines are busy?

(2) What is the probability that atleast two of them are busy? (8)

(ii) Describe Gamma distribution, Obtain its moment generating function. Hence compute its mean and variance. (8)

12. (a) X is a normal random variable with mean 1 and variance 4. Find the density function of Y where $Y = 2X^2 + 1$. (16)

Or

(b) (i) The joint pdf of X and Y is given by $f(x,y) = e^{-(x+y)}, x > 0, y > 0$. Find the probability density function of $U = \frac{X+Y}{2}$. (16)

13. (a) Discuss the stationarity of the random process $X(t) = A\cos(\omega_0 t + \theta)$ if A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (16)

Or

(b) (i) Define random telegraph signal process. Prove

(a) $P[X(t) = 1] = 1/2 = P[X(t) = -1]$, for all $t > 0$

(b) $E[X(t)] = 0$ and $Var[X(t)] = 1$ (8)

(ii) A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find

(1) the probability that he takes a train on the third day

(2) the probability that he drives to work in the long run. (8)

14. (a) State and Prove Wiener-Khintchine theorem, and hence find the power Spectral density of a WSS process *X(t)* which has an autocorrelation

$$R_{xx}(\tau) = A_0\left[1 - \frac{|\tau|}{T}\right], \quad -T \le \tau \le T. \quad (16)$$

Or

(b) (i) If $\{X(t)\}$ is a WSS process with auto correlation function $R_{XX}(\tau)$ and if $Y(t) = X(t+a) - X(t-a)$, show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$. (8)

(ii) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$. (8)

15. (a) Given the power spectral density of the continuous process, $\dfrac{\omega^2 + 2}{\omega^4 + 13\omega^2 + 36}$. Find the mean square value of the process. (16)

Or

(b) (i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ *and* $S_{YY}(\omega)$ are the power spectral density functions of the input *X(t)* and the output *Y(t)* and $H(\omega)$ is the system transfer function. (8)

(ii) The input to the RC filter is a white noise process with ACF $R_{XX}(\tau) = \dfrac{N_0}{2}\delta(\tau)$. If the frequency response $H(\omega) = \dfrac{1}{1 + j\omega RC}$, find the autocorrelation and the mean-square value of the output process Y(t). (8)