Reg. No. : | | | | | | | | | | |

Question Paper Code: 33021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. State the Dirichlet's conditions for the existence of a Fourier series.
2. State the conditions for $f(x)$ to have Fourier series expansion.
3. Find Fourier Sine Transform of $\frac{1}{x}$.
4. Define Fourier Integral theorem.
5. Find the Z-transform of a^n.
6. Find $Z[\frac{1}{n(n+1)}]$.
7. State initial and final value theorems on z - transform.
8. Write the appropriate solution of the one dimensional heat flow equation.
9. Write down the implicit formula to solve one dimensional heat flow equation.
10. State Liebmann's iteration process formula.

PART - B (5 x 16 = 80 Marks)

11. (a) Expand the function $f(x) = \sin x, 0 < x < \pi$ in a Fourier cosine series. (16)

Or

(b) (i) Find the Half range cosine series for $y = x$ in $(0, l)$ and hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots \infty$. (8)

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

12. (a) (i) Find the Fourier cosine transform of e^{-4x} and hence find the values of $\int_0^\infty \frac{\cos 2x}{x^2+16} dx$ and $\int_0^\infty \frac{x \sin 2x}{x^2+16} dx$ (8)

(ii) Find the Fourier cosine transform of $f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \pi < x < \infty \end{cases}$ (8)

Or

(b) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & : |x| < 1 \\ 0 & : otherwise \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4} dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x\, e^{-x^2}$. (8)

13. (a) (i) Using Convolution theorem, evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$ (8)

(ii) Using Z-transform solve the equation $u_{n+2} - 5u_{n+1} + 6u_n = 4^n, u(0) = 0$ and $u(1) = 1$. (8)

Or

(b) (i) State and prove initial and final value theorem on Z- transform. (8)

(ii) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by using method of Partial fraction. (8)

14. (a) A tightly stretched string of length l with fixed ends is initially in its equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3\left(\pi \frac{x}{l}\right)$. Find the displacement $y(x,t)$. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite length. If the temperature at short edge $y = 0$ is given by $u = \begin{cases} 20\,x & : 0 \le x \le 5 \\ 20(10-x) : & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at 0°C. Find the steady state temperature at any point of the plate. (16)

15. (a) Solve $\nabla^2 u = -10\left(x^2 + y^2 + 10\right)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $u = 0$ on the boundary and mesh length 1 unit. (16)

Or

(b) (i) Solve, by Crank-Nicholson method, the equation $u_{xx} = u_t$ subject to $u(x,0) = 0, u(0,t) = 0 \; and \; u(1,t) = t$ for two time steps by taking $h = 0.25$. (8)

` (ii) Evaluate the pivotal values of the following equation taking $h = 1$ and up to one half of the period of the oscillation $16u_{xx} = u_{tt}$ given $u(0,t) = 0$, $u(5,t) = 0$, $u(x,0) = x^2(5-x) \; and \; u_t(x,0) = 0$. (8)