Reg. No. : | | | | | | | | | | |

Question Paper Code: 51002

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2019

First Semester

Civil Engineering

01UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. State Cayley – Hamilton theorem and its uses.

2. Prove that, if A is orthogonal then A^T and A^{-1} are orthogonal.

3. Find the center and radius of the sphere $3(x^2+y^2+z^2)-2x-3y-4z-22=0$.

4. Define the right circular cylinder.

5. Find the curvature of the curve $2x^2+2y^2+5x-2y+1=0$.

6. Find the radius of curvature for $y = e^x$ at the point where it cuts the Y- axis (or) at x=0.

7. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

8. If $x = r\cos\theta$ and $y = r\sin\theta$, then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

9. Evaluate $\int_0^1\int_0^{x^2} (x^2 + y^2)dydx$.

10. Evaluate $\int_0^1\int_0^2\int_0^3 xy^2z\ dzdydx$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Eigen values and Eigenvectors of the matrix $A=\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem find A^4 and A^{-1} when $A=\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ to canonical form by orthogonal reduction. Also find the nature of the quadratic form. (16)

12. (a) (i) Find the center, radius and area of the circle $x^2+y^2+z^2-2x-4y-6z-2=0$, $x+2y+2z=20$. (8)

(ii) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$

$2x+3y+4z=8$; is a great circle. (8)

Or

(b) (i) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (8)

(ii) Find the equation of the right circular cylinder whose axis is the line $x = 2y = -z$ and radius 4. (8)

13. (a) (i) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. (8)

(ii) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$. (8)

Or

(b) (i) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters 'a' and 'b' are connected by the relation $a+b = c$. (8)

(ii) Find the envelope of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, considering it as the envelope of normals. (8)

14. (a) If $u = 2xy$, $\vartheta=x^2-y^2$ where if $x=r\cos\theta$, $y=r\sin\theta$ find $\frac{\partial(u,\vartheta)}{\partial(r,\theta)}$. (16)

Or

(b) (i) Examine $f(x,y)=x^3+y^3-12x-3y+20$ for its extreme values. (8)

(ii) Find the dimensions of the rectangular box without a top of maximum capacity, whose surface is 108 *sq.cm*.. (8)

15. (a) Change the order of the integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\ dxdy$. (16)

Or

(b) (i) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\ dx\ dy$ by changing into polar coordinates. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (8)
