Reg. No. : | | | | | | | | | | |

Question Paper Code: 31401

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2016

Fourth Semester

Computer Science and Engineering

01UMA421 - APPLIED STATISTICS AND QUEUEING NETWORKS

(Common to Information Technology)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 2 = 20 Marks)

1. A card is drawn from a well shuffled pack of playing cards. What is the probability that it is either a spade or an ace?

2. If X is a uniform random variable in (-2, 2), find the p.d.f of X and Var(X).

3. The joint p.d.f of the two dimensional random variable (X, Y) is given by $f(x,y)=\frac{8xy}{9}$, $1\le x\le y\le 2$. Find the marginal density function x and y.

4. Show that $Cov^2(x,y)\le Var(x).Var(y)$.

5. What is the aim of the design of experiments?

6. Write down the ANOVA table for one way classification.

7. Define a steady state condition.

8. Write Little's formula.

9. Define series queues. Give examples.

10. Define Open and closed queuing networks.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A continuous random variable X has p.d.f $f(x) = K x^2 e^{-x}, x \geq 0$. Find the r^{th} moment of X about the origin. Hence find mean and variance of X. (8)

(ii) In a component manufacturing industry, there is a small probability of 1/500 for any component to be defective. The components are supplied in packs of 10. Use Poisson distribution to calculate the approximate number of packets containing (a) no defective (b) two defective components in a consignment of 10000 packets. (8)

Or

(b) (i) A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B. (8)

(ii) Find the moment generating function of the geometric distribution and hence find its mean and variance. (8)

12. (a) (i) The random variable [X, Y] has the following joint p.d.f

$$f(x,y) = \begin{cases} \dfrac{x+y}{2}, & 0 \leq x \leq 2,\ 0 \leq y \leq 2 \\ 0 & ,otherwise \end{cases}$$

Obtain the marginal distribution of X and Y and compute co variance [X, Y]. (8)

(ii) 20 dice are thrown. Find approximately the probability that the sum obtained is between 65 and 75 using central limit theorem. (8)

Or

(b) (i) Obtain the equation of the lines of regression for the following data (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(ii) The joint probability mass function of X and Y is given below

y \ x	-1	1
0	$\frac{1}{8}$	$\frac{3}{8}$
1	$\frac{2}{8}$	$\frac{2}{8}$

Find correlation coefficient of (X, Y). (8)

13. (a) An experiment was designed to study the performance of four different detergents for cleaning fuel injectors. The following cleanness readings were obtained with specially designed equipment for 12 tanks of gas distributed over three different models of engines:

	Engine 1	Engine 2	Engine 3
Detergent *A*	45	43	51
Detergent *B*	47	46	52
Detergent *C*	48	50	55
Detergent *D*	42	37	49

Perform the ANOVA and test at 0.01 level of significance whether there are differences in the detergents or in the engines. (16)

Or

(b) Five varieties of wheat *A*, *B*, *C*, *D* and *E* were tried. The gross size of the plot was 18 *feet* $\times$ 22 *feet*, the net plot being 14 *feet* $\times$ 18 *feet*. Thus the whole experiment occupied an area 90*feet* $\times$110 *feet*. The plan, the varieties shown in each plot and yields obtained in *kg*. are given in the following table.

*B*90	*E*80	*C*134	*A*112	*D*92
*E*85	*D*84	*B*70	*C*141	*A*82
*C*110	*A*90	*D*87	*B*84	*E*69
*A*81	*C*125	*E*85	*D*76	*B*72
*D*82	*B*60	*A*94	*E*85	*C*88

Carry out an analysis of variance. What inference can you draw from the data given? (16)

14. (a) (i) A television repairman finds that the time spend on his job has an exponential distribution with mean 30 minutes. If the repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of average set just brought? (8)

(ii) A super market has 2 girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 an hour. (a)What is the probability of having to wait for service? (b) What is the expected percentage of idle time for each girl? (8)

Or

(b) (i) Consider a single server queuing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units in the system is two. Find the steady state probability distribution of the number of calling units in the system and the expected number of calling units in the system. (8)

(ii) A barber shop has two barbers and three chairs for customers. Assume that customers arrive in a Poisson fashion at a rate of 5 per hour, and that each barber services customers according to an exponential distribution with mean of 15 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave. What is the probability that the shop is empty? What is the expected number of customers in the shop? (8)

15. (a) Derive the Pollaczek - Khintchine formula. (16)

Or

(b) (i) There are two salesmen in a ration shop, one in charge of billing and receiving payment and the other in charge of weighing and delivering the items. Due to limited availability of space, only one customer is allowed to enter the shop, that too when the billing clerk is free. The customer who has finished his billing job has to wait there until the delivery section becomes free. If customers arrive in accordance with a Poisson process at rate 1 and the service times of two clerks are independent and have exponential rates of 3 and 2. Find (a) the proportion of customers who enter the ration shop (b) the average number of customers in the shop. (8)

(ii) In a network of service stations 1, 2, 3 customers arrive at 1, 2, 3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective rates 10, 50, 100. A customer completing service at station one is equally likely to (i) go to station 2 (ii) go to station 3 or (iii) leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go to station 2 or leave the system. (a) What is the average number of customers in the system, consisting of all the three station? (b) What is the average time a customer spends in the system? (8)
