Reg. No. : | | | | | | | | | | |

Question Paper Code: 51102

B.E. / B.Tech. DEGREE EXAMINATION, JUNE 2016

First Semester

Civil Engineering

15UMA102 – ENGINEERING MATHEMATICS - I

(Common to ALL Branches)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\frac{d}{dx}\left(x e^{x}\right)$ is equal to

   (a) $x e^{x} + e^{x}$ (b) $x e^{x}$ (c) e^{x} (d) x

2. $\frac{d}{dx}\left(\frac{u}{v}\right)$ is equal to

   (a) $\frac{v-u}{v^{2}}$ (b) $\frac{v u' - u v'}{v^{2}}$ (c) $v' u'$ (d) $u' - v'$

3. If $f(x, y)$ is a homogeneous function of degree n in x and y then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to

   (a) f (b) n (c) $n f$ (d) $\frac{n}{f}$

4. $\frac{\partial z}{\partial x}.\frac{dx}{dt} + \frac{\partial z}{\partial y}.\frac{dy}{dt}$ is equal to

   (a) $\frac{dz}{dt}$ (b) $\frac{\partial z}{\partial t}$ (c) $\frac{dt}{dz}$ (d) $\frac{\partial t}{\partial z}$

5. $\int \frac{1}{x}\,dx$ is equal to

(a) $\frac{x^2}{2}+c$ (b) $x+c$ (c) $\log x+c$ (d) e^x+c

6. $\int_0^2 x^2\,dx$ is equal to

(a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{6}{3}$ (d) $\frac{2}{3}$

7. $\int_1^2\int_2^5 xy\,dx\,dy$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{5}{6}$ (c) $\frac{18}{19}$ (d) $\frac{63}{4}$

8. Area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) πa (b) πb (c) πab (d) $\frac{\pi}{ab}$

9. If A is an orthogonal matrix then

(a) $|A|=0$ (b) A is singular (c) $A^2=I$ (d) $A^T=A^{-1}$

10. If $|A|=0$, then one of the Eigen values is

(a) 0 (b) 1 (c) 2 (d) -1

PART - B (5 x 2 = 10 Marks)

11. If charge $q=A\cos kt+B\sin kt$ find the convert i.

12. Prove that $JJ^1=1$.

13. Find $\int x e^x\,dx$.

14. Change the order of integration in $I=\int_0^2\int_0^x f(x,y)\,dy\,dx$.

15. State Cayley – Hamilton theorem.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) If $y = \tan^{-1} x$ prove that $(1+x^2)y_2 + 2xy_1 = 0$ and deduce that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$ (8)

(ii) Find the Maclaurian series for $\sin x$. (8)

Or

(b) An e.m.f $E \sin pt$ is applied at $t = 0$ to a circuit containing a capacitance C and inductance L. The charge q is given by $L\frac{d^2q}{dt^2} + \frac{q}{C} = E \sin pt$. If $p^2 = \frac{1}{LC}$ and initially the current and charge are zero. Find the current i at a time t. (16)

17. (a) (i) If $u = f(x-y, y-z, z-x)$ Prove $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (8)

(ii) Find the maxima and minima of $x^3 y^2 (1-x-y)$. (8)

Or

(b) (i) If $x^y + y^x = a^k$ *find* $\frac{dy}{dx}$. (8)

(ii) Obtain the dimensions of a rectangular box without top of maximum capacity given the surface area as 432 square meters. (8)

18. (a) (i) Evaluate $\int \frac{e^{\sin_x^{-1}}}{\sqrt{1-x^2}} dx$ (8)

(ii) Evaluate $\int \sin^n x \, dx$. (8)

Or

(b) (i) Prove that $\beta(m,n) = \int_0^\alpha \frac{y^{m-1}}{(1+y)^{m+n}} dy$ (8)

(ii) Evaluate $\int_0^\alpha \frac{dx}{1+x^4}$ (8)

19. (a) (i) Find the area common to $y^2 = 4ax$ *and* $x^2 = 4ay$ using double integration. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration. (8)

Or

(b) (i) Change the order of integration $I = \int_0^a \int_y^a \frac{x}{x^2 + y^2} dxdy$ and hence evaluate. (8)

(ii) Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)

20. (a) (i) Using Cayley-Hamilton theorem, find A^{-1} *if* $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$. (8)

(ii) Find Eigen values and Eigen vector of the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$. (8)

Or

(b) Reduce $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. (16)
