**Reg. No. :** | | | | | | | | | | |

**Question Paper Code: 31404**

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2016

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical tables may be permitted)

PART A - (10 x 2 = 20 Marks)

1. If a random variable $X$ has the moment generating function $M(t) = \frac{3}{3-t}$ , obtain the standard deviation of the variable $X$.

2. Two dice are thrown 720 times; find the average number of times in which the number on the first dice exceeds the number on the second dice.

3. he joint probability density function of the random variable *(X ,Y)* is given by $f(x, y) = k\,x\,y\;e^{-(x^2 + y^2)}, x > 0, y > 0$. Find the value of "$k$" and prove also that $X$ and $Y$ are independent.

4. State the equations of the two regression lines.

5. Classify the random processes.

6. Show that the Poisson process is a Markov process.

7. State any two properties of cross correlation function.

8. Show that the spectral density function of a real random process is an even function.

9. Define a system. When it is called linear system?

10. Prove that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A random variable $X$ has the following probability distribution:

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(x) | 0 | $k$ | $2k$ | $3k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2 + k$ |

Find (i) the value of $k$, (ii) $P(1.5 < X < 4.5 \,/\, X > 2)$ and (iii) the smallest value of $\lambda$ for which $P(X \leq \lambda) > \frac{1}{2}$. (8)

(ii) Show that sum of any two Poisson variables is again a Poisson variate. (8)

Or

(b) (i) Find the mean and variance of Binomial distribution. (8)

(ii) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 *kms*. Find the probabilities that one of these tires will last (i) at least 20,000 *kms*, and (ii) at most 30,000 *kms*. (8)

12. (a) $X$ is a normal random variable with mean 1 and variance 4. Find the density function of Y where $Y = 2X^2 + 1$. (16)

Or

(b) (i) Find the marginal density functions of $X$ and $Y$ from the joint density function of $X$ and $Y$

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \leq x, y \leq 1 \\ 0, & elsewhere \end{cases}$$ (8)

(ii) Obtain the lines of regression and find the coefficient of correlation from the following. (8)

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

13. (a) Prove that the random process *[X(t)]* with constant mean is mean ergodic, if $\lim_{T\to\infty} \int_{-T}^{T} \int_{-T}^{T} \frac{C(t_1, t_2)}{4T^2}\, dt_1 dt_2 = 0$. (16)

Or

(b) (i) Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (i) exactly 4 customers arrive and (ii) more than 4 customers arrive. (8)

(ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now, suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run. (8)

14. (a) State and prove Wiener-Khinchine theorem. (16)

Or

(b) (i) Two random processes *X(t)* and *Y(t)* are defined by $X(t) = A\, Cos\, \omega_0 t + B \sin \omega_0 t$ and $Y(t) = B\, Cos\, \omega_0 t - A \sin \omega_0 t$. Show that *X(t)* and *Y(t)* are jointly wide-sense stationary, if *A* and *B* are uncorrelated random variables with zero means and the same variances and $\omega_0$ is a constant. (8)

(ii) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\gamma|\tau|}$. Determine the power density spectrum of the random telegraph signal. (8)

15. (a) (i) Prove that if the input to a time-invariant , stable linear system is a WSS process, then the output will also be a WSS process. (8)

(ii) If the input *x*(t) and the output *y*(t) are connected by the differential equation $T\frac{dy(t)}{dt} + y(t) = x(t)$, prove that they can be related by means of a convolution type integral, assuming that *x*(t) and *y*(t) are zero for $t \le 0$. (8)

Or

(b) X(t) is input voltage to a circuit (system) and *Y(t)* is the output voltage. *{X(t)}* is a stationary random process with mean zero and autocorrelation function $R(\tau) = e^{-\alpha|\tau|}$. Find the mean, auto correlation function and spectral density function of the output *{Y(t)}*, if the power transfer function is $H(\omega) = \frac{R}{R+iL\omega}$. (16)

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