Reg. No. : | | | | | | | | | | |

Question Paper Code: 31501

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2016

Fifth Semester

Computer Science and Engineering

01UMA521 - DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulation 2013)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Define tautology.

2. Construct the truth table for $7q \wedge (p \rightarrow q)$.

3. State Pigeonhole principal.

4. Find the generating function for the sequence 0, 0, 0, 6, -6, 6, …

5. State Handshaking theorem.

6. Define spanning tree.

7. Define semi group.

8. State the condition for a subgroup of a group to be normal.

9. Let $A = \{0,2,5,10,11,15\}$, draw a Hasse diagram for $\{A, \leq\}$.

10. Define bounded lattice.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Obtain the principal disjunctive normal form and the principal conjuctive normal form of statement $(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$. (8)

(ii) Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows from the premises $\forall x(P(x) \to Q(x))$ and $\exists y P(y)$. (8)

Or

(b) (i) Derive $p \to (q \to s)$ using the CP–rule (if necessary) from the premises $p \to (q \to r)$ and $q \to r(r \to s)$. (8)

(ii) Show that the premises "one student in this class knows how to write programs in JAVA "and "Everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "someone in this class can get a high-paying job". (8)

12. (a) (i) Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3, for $n \geq 1$. (8)

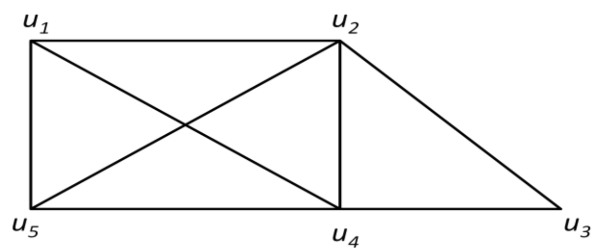
(ii) Solve the recurrence relation $a_n = 2a_{n-1} + 2^n; a_0 = 2$. (8)

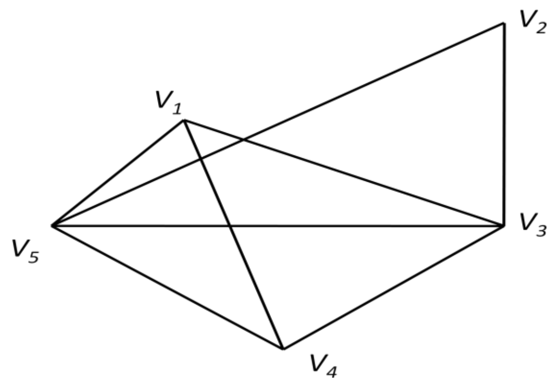
Or

(b) (i) Use the method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1; n \geq 1$, given that $a_0 = 1$. (8)

(ii) There are 250 students in an engineering college. Of these 188 have taken a course in Fortran, 100 have taken a course in *C* and 35 have taken a course in Java. Further 88 have taken courses in both Fortran and *C*. 23 have taken courses in both *C* and Java and 29 have taken courses in both Fortran and Java. If 19 of these students have taken all the three courses, how many of these 250 students have not taken a course in any of these three programming languages? (8)

13. (a) (i) Determine whether the following pairs of graphs are isomorphic. (8)





(ii) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)\ (n-k+1)}{2}$. (8)

Or

(b) (i) Give an example of a graph which contains

(1) an Eulerian circuit that is also a Hamiltonian circuit

(2) an Eulerian circuit and a Hamiltonian circuit that are distinct

(3) an Eulerian circuit, but not a Hamiltonian circuit

(4) a Hamiltonian circuit, but not an Eulerian circuit (8)

(ii) Prove that the number of edges in a bipartite graph with n vertices is at most $\left(\frac{n^2}{2}\right)$. (8)

14. (a) (i) Prove that the intersection of two subgroups of a group G is also a subgroup of G. (8)

(ii) State and prove Lagrange's theorem. (8)

Or

(b) (i) State and prove fundamental theorem homomorphism. (8)

(ii) If H is a normal subgroup of G and K is a subgroup of G such that $H \subseteq K \subseteq G$, show that H is a normal subgroup of K also. (8)

15. (a) (i) State and prove distributive inequality on lattices. (8)

(ii) State and prove De Morgon's law for Boolean algebra. (8)

Or

(b) (i) Prove that if $\{L, \leq\}$ is a lattice in which $\vee$ and $\wedge$ denote the operations of join and meet respectively, then for $a, b \in L$, $a \leq b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a$. (8)

(ii) Simplify the following expressions using Boolean algebra:

(1) $a'b(a' + c) + ab'(b' + c)$

(2) $a + a'bc' + (b + c)'$

(8)