Reg. No. : | | | | | | | | | | |

Question Paper Code: 41301

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2016

Third Semester

Civil Engineering

14UMA321-TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2014)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If $f(x)$ is even in $(-\pi, \pi)$ then the Fourier Coefficient $b_n =$ ___________

   (a) 1 (b) 0 (c) π (d) $-\pi$

2. R.M.S value of $f(x) = x$ in (-1, 1) is____________

   (a) 0 (b) 1 (c) $\frac{1}{3}$ (d) $\sqrt{\frac{1}{3}}$

3. If $F(f(x)) = F(s)$ and $F(g(x)) = G(s)$ then $F(f(x) * g(x)) =$ _______

   (a) $F(s)G(s)$ (b) $F(s)*G(s)$ (c) $f(s)*g(s)$ (d) $F(x)*G(x)$

4. If $F(f(x)) = F(s)$ then $F(f(x-a)) =$ _______

   (a) $e^{aix}F(s)$ (b) $e^{-ais}F(s)$ (c) $e^{ais}F(s)$ (d) $e^{-aix}F(s)$

5. $\lim_{z \to 1} (z-1)\, F(z) =$ ______

(a) $f(1)$ (b) $F(\infty)$ (c) $f(\infty)$ (d) $f(0)$

6. Z(1) = ______

(a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{1}{z+1}$ (d) $\frac{1}{z-1}$

7. The two-dimensional heat equation is ______

(a) $\nabla^2 u = 0$ (b) $\nabla^2 u = u_{tt}$

(c) $u_{xx} + u_{yy} = 0$ (d) $u_t = a^2 \left(u_{xx} + u_{yy}\right)$

8. Classify the partial differential equation $4u_{xx} = u_t$

(a) Hyperbolic (b) Parabolic (c) Elliptic (d) Poisson

9. The finite difference approximation to $y_i' =$ ____________

(a) $\frac{y_{i+1} - y_{i-1}}{h}$ (b) $\frac{y_{i+1} + y_{i-1}}{h}$ (c) $\frac{y_{i+1} - y_{i-1}}{2h}$ (d) $\frac{y_{i+1} + y_{i-1}}{2h}$

10. The Laplace equation is ______

(a) $\nabla^2 u = \nabla x$ (b) $\nabla^2 u = f\left(x, y\right)$

(c) $\nabla^2 u - f\left(x, y\right) = 0$ (d) $\nabla^2 u = 0$

PART - B (5 x 2 = 10 Marks)

11. Find the Fourier constant a_0 for $f\left(x\right) = k$ in $\left(0, 2\pi\right)$.

12. State the Fourier integral theorem.

13. Find the Z – transform of *f(n)= n*.

14. Write all possible solutions of the two-dimensional heat equation in steady state.

15. Write down the standard five-point formula to solve the Laplace equation.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the Fourier series for $f(x) = 1 + x + x^2$ in $(-\pi, \pi)$. Deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\infty = \frac{\pi^2}{6}$ (12)

(ii) Find the half range sine series for $f(x) = x$ in $(0, \pi)$. (4)

Or

(b) (i) Find the cosine series for $f(x) = \pi x - x^2$ in $(0, \pi)$ and show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots = \frac{\pi^4}{90}$. (8)

(ii) Find Fourier series up to second harmonic for the following data: (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1

17. (a) (i) Evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2 + a^2)(x^2 + b^2)}$. (8)

(ii) Find Fourier cosine transform of $e^{-a^2x^2}$. (8)

Or

(b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence deduce the value of

(i) $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2}\,ds$ (ii) $\int_0^\infty \left[\frac{\sin s - s\cos s}{s^3}\right]^2 ds$ (16)

18. (a) (i) Find the inverse Z – transform of $\frac{10z}{(z-1)(z-2)}$. (8)

(ii) State and prove convolution theorem on Z – Transform. (8)

Or

(b) (i) Solve using $Y_{n+2} - 3Y_{n+1} + 2Y_n = 2^n$ given that $Y_0 = 0$ and $Y_1 = 0$ using Z – transform. (10)

(ii) Find Z- transform of $\frac{1}{(n+1)(n+2)}$. (6)

19. (a) If a string of length l is initially at rest in its equilibrium position and each of its point is given a velocity $v_0 \sin^3 \left(\pi x / l\right)$, $0 < x < l$, find the displacement of the string $y(x, t)$. (16)

Or

(b) A metal bar 20cm long, with insulated sides, has its ends A and B kept at 30ºC and 90ºC respectively until steady state conditions prevail. The temperature at each end is suddenly raised to 0ºC and kept so. Find the subsequent temperature at any time of the bar at any time. (16)

20. (a) Solve the Poisson's equations $\nabla^2 u = -81xy$, $0 < x < 1$, $0 < y < 1$, $h = \frac{1}{3}$, $u(0, y) = u(x,0) = 0$, $u(1, y) = u(x, 1) = 100$. (16)

Or

(b) (i) Solve $u_{xx} = 32\, u_t$ with h = 0.25 for $t > 0$; $0 < x < 1$ and $u(x,0) = u(0, t) = 0$; $u(1, t) = t$. Tabulate u up to $t = 5$ sec using Bender-Schmidt formula. (8)

(ii) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < 1$, $t > 0$, give $u(x, 0) = 0$, $u(0, t) = 0$, $u_t(x, 0) = 0$ and $u(1, t) = 100 \sin \pi t$ Compute u for 4 times steps with $h = 0.25$. (8)
