Reg. No. : | | | | | | | | | | |

Question Paper Code: 31301

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2016

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Write down the Dirichlet's conditions for a function to be expanded as a Fourier series.
2. Find the Fourier constant b_n for $f(x)=x\sin x$ in $(-\pi,\pi)$.
3. State Fourier integral theorem.
4. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$
5. Find $Z[n]$.
6. State the convolution theorem of Z-transforms.
7. Write the three possible solutions of one dimensional wave equation.
8. Classify the PDE $f_{xx} - 2f_{xy} = 0$.
9. Write the difference scheme for solving the Laplace's equation.
10. Write the Bender Schmidt's recurrence relation one dimensional heat equation.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Fourier series of $f(x) = \begin{cases} 1 & in \quad (0,\pi) \\ 2 & in \quad (\pi, 2\pi) \end{cases}$

Hence find the sum of the series $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} ..... \infty$ (8)

(ii) Obtain the Fourier series of the function $f(x) = \begin{cases} 1+x & in \quad 0 < x < \pi \\ x-1 & in \quad -\pi < x < 0 \end{cases}$. (8)

Or

(b) (i) Find the Half range cosine series for $y = x$ in $(0, l)$ and hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...\infty$. (8)

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

12. (a) (i) Prove that the function $f(x) = e^{-\frac{x^2}{2}}$ is self reciprocal under Fourier transform. (8)

(ii) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$ (8)

Or

(b) Find Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & |x| \le a \\ 0 & |x| > a \end{cases}$ Using Parseval's identity, prove that $\int_0^\infty \left(\frac{sins - scoss}{s^3}\right)^2 ds = \frac{\pi}{15}$. (16)

13. (a) (i) Find $Z\left[\frac{2n+3}{n\,(n\,+1)}\right]$. (8)

(ii) Find $Z^{-1}\left[\frac{Z}{Z^2 + 11Z + 24}\right]$. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ using convolution theorem. (8)

(ii) Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0$, $y_0 = -1, y_1 = 2$. (8)

14. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at $t = 0$. Find the displacement of any point at a distance 'x' at any time 't'. (16)

Or

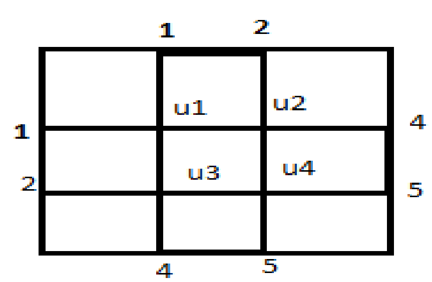
(b) A infinitely long rectangular plate with insulated surfaces is $5cm$ wide. If the temperature at the short edge $x = 0$ is given $u(0, y) = 3y$ and the two long edges as well as the other short edge are kept at 0^0C, find the steady state temperature distribution in the plate. (16)

15. (a) (i) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ *subject to* $u(x,0) = \sin \pi x$; $0 < x < 1$; $u(0, t) = 0$, $u(1, t) = 0$, compute u upto t=0.1 taking $h = 0.2$, by using Bender-Schmidt recurrence equation. (8)

(ii) Solve by Crank-Nicolson's method $\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2}$; $0 \leq x \leq 1$; subject to $t > 0$, $u(x,0) = 0, u(0,t) = 0; u(1,t) = 100t$. Compute u for one step with $h = 1/4$. (8)

Or

(b) (i) Using Leibmann iteration, solve the Laplace's equation $\nabla^2 u = 0$ for the following square mesh with boundary values as shown in below figure (only four iterations).



(8)

(ii) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0, t) = u(4, t) = u_t(x, 0) = 0$ and $u(x, 0) = x(4-x)$ taking $h = 1$. (8)