Reg. No. : | | | | | | | | | | |

Question Paper Code: 51202

B.E. / B.Tech. DEGREE EXAMINATION, JUNE 2016

Second Semester

Civil Engineering

15UMA202 – ENGINEERING MATHEMATICS-II

(Common to ALL Branches)

(Regulation 2015)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Find the complementary function for the differential function $(D^2 - 2D + 5)\,y = 0$

   (a) $e^{x}\,(A\cos x + B\sin x)$ (b) $e^{2x}\,(A\cos 2x + B\sin 2x)$
   (c) $e^{-x}\,(A\cos x + B\sin x)$ (d) $e^{x}\,(A\cos 2x + B\sin 2x)$

2. What is the particular integral of the solution of $(D^2 + D + 1)\,y = e^{x}$?

   (a) $\frac{1}{3}e^{x}$ (b) $\frac{1}{4}e^{x}$ (c) $-\frac{1}{2}e^{x}$ (d) $-\frac{1}{4}e^{x}$

3. $\nabla^2\,(r^n) = n(n+1)\,r^{n-2}$ reduces to Laplace equation when n is

   (a) 0 (b) 1 (c) -1 (d) 2

4. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then div $\vec{r}$ is

   (a) 1 (b) 0 (c) 2 (d) 3

5. If $f(z) = \frac{1}{z^2 + 1}$ is analytic everywhere except at

(a) $z = \pm 1$ (b) $z = \pm i$ (c) $z = i$ (d) $z = -i$

6. The analytic function with constant real part will be

(a) Analytic (b) Constant (c) Real (d) Zero

7. The value of $\int_c \frac{dz}{2z-3}$ where C is the circle $|z|=1$.

(a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) πi

8. If $z = 1$ is a simple pole of $f(z) = \frac{z}{z^2 - 1}$; the [Residue of $f(z)]_{z=1}$ is

(a) 0 (b) 0.5 (c) 1 (d) 1.5

9. $L(t^3)$ is equal to

(a) $\frac{3!}{s^3}$ (b) $\frac{3!}{s^4}$ (c) $\frac{3!}{t^4}$ (d) $\frac{4!}{s^3}$

10. $L^{-1}\left[\frac{1}{s^2 + 4}\right]$ is equal to

(a) $\cos 2t$ (b) $\sin 2t$ (c) $\frac{1}{2}\cos 2t$ (d) $\frac{1}{2}\sin 2t$

PART - B (5 x 2 = 10 Marks)

11. Reduce $(x^2 D^2 - xD + 7)y = 2x$ into a differential equation with constant coefficients.

12. State Stokes theorem.

13. Prove that the function $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic.

14. Define Cauchy's residue theorem.

15. State the conditions under which Laplace transform exists.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Solve $(D^2 - 3D + 2)\, y = \sin 2x + x.$ (8)

(ii) Solve $(x^2 D^2 - 3xD + 4)y = x^2 + \cos(\log x).$ (8)

Or

(b) (i) Solve $(x + 2)^2 \dfrac{d^2 y}{dx^2} - (x + 2)\dfrac{dy}{dx} + y = 3x + 4.$ (8)

(ii) Solve $\dfrac{d^2 y}{dx^2} + 4y = \sec 2x$ by the method of variation of parameters. (8)

17. (a) (i) Determine a and b so that the vector $\overline{F} = 3x^2 y\bar{i} + (ax^3 - 2yz^2)\bar{j} + (3z^2 + by^2 z)\bar{k}$ is irrotational and hence find its scalar potential Φ such that $\overline{F} = \nabla\Phi$. (8)

(ii) Using Green's theorem in plane for $\int_C (xy + y^2)\, dx + x^2\, dy$ where C is the region bounded between $y = x^2$ and $y = x.$ (8)

Or

(b) Verify Gauss divergence theorem for $\overline{F} = x^2\,\bar{i} + y^2\,\bar{j} + z^2\,\bar{k}$ taken over the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1.$ (16)

18. (a) (i) State and prove any two properties of analytic functions. (8)

(ii) Construct the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y).$ (8)

Or

(b) (i) Find the image of the circle $|z - 1| = 1$ of the z-plane under the mapping $w = \dfrac{1}{z}.$ (8)

(ii) Find the bilinear transformation that maps the points $z = (-i,\ 0,\ i)$ into the points $w = (-1,\ i,\ 1).$ (8)

19. (a) (i) Evaluate $\int_C \frac{z+1}{z^2+2z+4}\,dz$ where C is the circle $|z+1+i| = 2$. (8)

(ii) Find the Laurent's series expansion of the function $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$. (8)

Or

(b) (i) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$, where C is the circle $|z| = \frac{3}{2}$ by Cauchy's residues formula. (8)

(ii) Apply contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$. (8)

20. (a) (i) Find $L\left(e^{-2t}\, t^5\right)$ (4)

(ii) Find $L\left(\frac{1-\cos at}{t}\right)$ (4)

(iii) Find Laplace transform of full sine wave rectifier

$$f(t) = \begin{cases} E\sin\omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases} \text{ and } f\left(t + \frac{2\pi}{\omega}\right) = f(t).$$ (8)

Or

(b) (i) Using Convolution theorem find $L^{-1}\left[\frac{s}{\left(s^2+a^2\right)^2}\right]$. (8)

(ii) Using Laplace transform solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4$ given that $y(0) = 2$ and $y'(0) = -3$. (8)