Reg. No. : | | | | | | | | | | |

Question Paper Code: 21002

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2014.

Second Semester

Civil Engineering

01UMA202 - ENGINEERING MATHEMATICS - II

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Solve $(x^2 D^2 + xD - 1)\, y = 0$.
2. Find the *P.I* of $(D^3 - 1)\, y = e^{2x}$.
3. State Green's theorem.
4. Prove that div $\vec{r} = 3$.
5. Check whether xy^2 is real part of an analytic function.
6. Find the fixed points of $w = \dfrac{3z-4}{z-1}$.
7. State Cauchy' s Integral Formula.
8. Expand $\dfrac{1}{z-2}$ at $z = 1$ in a Taylor's series.
9. Find $L[e^t \sin 2t]$.
10. Define existence conditions of Laplace transform.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Solve: $(2x + 3)^2 \frac{d^2y}{dx^2} - 2(2x + 3) \frac{dy}{dx} - 12y = 6x$. (8)

(ii) Solve $(D^2+4)\, y = x \sin x$. (8)

Or

(b) (i) Solve $(D^2+2D+5)\, y = e^{-x} \tan x$ by method of variation of parameter. (8)

(ii) Solve $\frac{dx}{dt} + y = \sin t$ and $\frac{dy}{dt} + x = \cos t$ given x = 2, y =0 when t = 0. (8)

12. (a) Verify divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where S is the surface formed by the planes x = 0, x = a, y = 0, y = b, z = 0 and z = c. (16)

Or

(b) Verify Stoke ' s theorem for $\overline{F} = y\vec{i} + z\vec{j} + x\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (16)

13. (a) (i) If $f(z) = u + iv$ is a regular function of z, then show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) | f(z)|^p = p^2 \, | f(z)|^{p-2} | f'(z)|^2 .$$ (8)

(ii) Find the bilinear transformation which maps x = 1, i , -1 respectively onto w = i, 0, -i. . (8)

Or

(b) (i) If $f(z) = u + iv$ an analytic function and $u - v = e^x (\cos y - \sin y)$ find $f(z)$ interms of z. . (8)

(ii) Find the image of $|z - 2i| = 2$, under the transformation $w = 1/z$. (8)

14. (a) (i) Evaluate $\int_c \frac{z+1}{z^2+2z+4} dz$, where C is the circle $|z + 1 + i| = 2$. (8)

(ii) Evaluate $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$. (8)

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Or

(b) (i) Find Laurent's series expansion of f(z) = $\frac{7z-2}{z(z-2)(z+1)}$ valid 1< |z+1|<3. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$. (8)

15. (a) (i) Solve by using Laplace transform $(D^2 + 1)\, y = 2e^t$, given that if $y(0) = 1, y'(0) = 2$. (8)

(ii) Verify initial value and final value theorem for the function $1+ e^{-2t}$. (8)

Or

(b) (i) Find the Laplace transform of the Half-wave rectifier function

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases} \text{, with } f\left(t + \frac{2\pi}{\omega}\right) = f(t) \ .$$ (8)

(ii) Using convolution theorem find $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$. (8)
