Reg. No. : | | | | | | | | | | |

Question Paper Code: 11002

B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2013.

First Semester

Civil Engineering

01UMA102 - ENGINEERING MATHEMATICS - I

(Common to All Branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Two of the Eigen values of 3×3 matrix where determinant value equal to 4 are -1 and 2. Find the 3rd Eigen value.

2. Prove that, if A is orthogonal then A^T and A^{-1} are orthogonal.

3. Find the equation of the sphere with centre (2, 3, 5) and touches the XoY – plane.

4. Define the right circular cylinder.

5. Find the curvature of the curve $2x^2+2y^2+5x-2y+1=0$.

6. Find the envelope of the straight lines represented by the equation $x \cos\alpha + y \sin\alpha = a \sec\alpha$,where α is the parameter.

7. If $W=f\left[\frac{x}{z},\frac{y}{z}\right]$, then prove that $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = 0$.

8. If $(\cos x)^y = (\sin y)^x$ find $\frac{dy}{dx}$.

9. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dy\, dx$.

10. Evaluate $\int_0^1 \int_0^2 \int_0^e dz\, dy\, dx$.

PART – B (5 X 16 = 80marks)

11. (a) (i) Find the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (8)

(ii) Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence evaluate

$A^8-5A^7+7A^6-3A^5+A^4-5A^3-8A^2+2A$. (8)

Or

(b) (i) Reduce the quadratic form $2xy+2yz+2zx$ to a canonical form by orthogonal reduction. Also find the rank, index, signature and nature of the quadratic form. (16)

12. (a) (i) Find the equations of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x+2y-z+5=0$. (8)

(ii) Find the centre and radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+17 = 0$. Also find the equation of the sphere having the above circle as great circle. (8)

Or

(b) (i) Find the equation of the cone whose vertex is the origin and guiding curve is $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{1}=1,\ x+y+z=1$. (8)

(ii) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$. (8)

13. (a) (i) For the curve $y=\frac{ax}{a+x}$, if P is the radius of curvature at any point (x,y), show that $\left(\frac{2P}{a}\right)^2 = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ (8)

(ii) Find the evolute of the parabola $x^2=4ay$. (8)

Or

(b) (i) Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$ where the parameters 'a' and 'b' are connected by the relation $a+b = c$. (8)

(ii) Find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, considering it as the envelope of normals. (8)

14. (a) (i) If $g(x, y) = k(u,\vartheta)$ where $u = x^2 - y^2$, $\vartheta = 2xy$ prove that

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2+y^2)\left[\frac{\partial^2 k}{\partial u^2} + \frac{\partial^2 k}{\partial \vartheta^2}\right]$$ (8)

(ii) If $u = 2xy$, $\vartheta = x^2 - y^2$ where if $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(u,\vartheta)}{\partial(r,\theta)}$. (8)

Or

(b) (i) Expand $e^x \cos y$ in powers of x and y as far as the terms of third degree. (8)

(ii) Find the dimensions of the rectangular box without a top of maximum capacity, whose surface is 108 *sq.cm*. (8)

15. (a) (i) Evaluate $\iint y \, dx \, dy$ over the region bounded by $x^2 = y$ and $x + y = 2$ in the positive quadrant. (8)

(ii) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} \, dx \, dy$ by changing the order of integration. (8)

Or

(b) (i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$ by changing into polar coordinates. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$ and

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)