Reg. No. : | | | | | | | | | | |

Question Paper Code: 12061

M.E. DEGREE EXAMINATION, DECEMBER 2013.

First Semester

STRUCTURAL ENGINEERING

01PMA125 - APPLIED MATHEMATICS

(Regulation 2013)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Prove that $L\left[\frac{\partial u}{\partial t}; s\right] = sU(x,s) - u(x,0)$, where $U(x,s) = L[u(x,t); s]$.

2. Find the Fourier transform of $\frac{\partial u}{\partial t}$.

3. State Poisson equation.

4. If f is a function of x and t, $0 < x < L, t > 0$ then prove that $F_s\left[\frac{\partial f}{\partial x}; n\right] = \frac{-n\pi}{L} F_c(n)$.

5. Find the Ostrogradsky equation for $I[z(x,y)] = \iint \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy$.

6. Find the extremals of the functional $\int_{-1}^{0} (12xy - y'^2)dx$; $y(-1) = 1$, $y(0) = 0$.

7. What is the use of Power method and when it is applicable?

8. Define Rayleigh quotient of a Hermition matrix.

9. Evaluate $\int_0^1 \frac{dx}{x^2}$ using Gaussian two point quadrature formula.

10. Define Monte-Corlo method.

PART -- B (5 x 16 = 80 Marks)

11. (a) Using Laplace transform method, solve PDE: $u_t = 3u_{xx}$
BCs: $u\left(\frac{\pi}{2}, t\right) = 0, u_x(0,t) = 0$, IC:$u(x,0) = 30\cos 5x$ (16)

Or

(b) Solve PDE: $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} + k,\ 0 \le x \le l, t > 0$
subject to BCs: $u(0,t) = u_x(l,t) = 0, t > 0$, ICs: $u(x,0) = u_t(x,0), 0 \le x \le l.$
(16)

12. (a) A uniform string of length L is stretched tightly between two fixed points at $x = 0$ and $x = L$. If it is displaced a small distance ε at a point $x = b, 0 < b < L$, and released from at time $t = 0$, find an expression for the displacement at subsequent times. (16)

Or

(b) Using the method of integral transform to solve the following potential problem in the semi infinite strip described by
PDE: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < \infty,\ 0 < y < a.$
subject to BCs: $u(x,0) = f(x), u(x,a) = 0,$
$u(x,y) = 0,\ 0 < x < \infty,\ 0 < y < a$ and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty.$ (16)

13. (a) (i) Find the extremal for $I = \int_0^{\frac{\pi}{2}} (y''^2 - y^2 + x^2)dx, y(0) = 1, y'(0) = 0, y(\frac{\pi}{2}) = 0, y'(\frac{\pi}{2}) = -1$
(10)

(ii) Find the extremal of the functional $I = \int_1^2 \frac{x^3}{y'^2} dx, y(1) = 1, y(2) = 4.$ (6)

Or

(b) Find an approximate solution to the problem of the minimum of functional $v[y(x)] = \int_0^1 (y'^2 - y^2 + 2xy)dx, y(0) = 0 = y(1)$ by Ritz method and compare it with the exact solution. (16)

.

14. (a) (i) Explain Power method with deflation. (6)

(ii) Obtained the characteristic polynomial of the matrix $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$ by Faddevv-Leverrier method. (10)

Or

(b) (i) Explain Faddevv-Leverrier method. (6)

(ii) Using Power method find all the Eigen values of $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. (10)

15. (a) (i) Evaluate $\int_{-\infty}^{\infty} e^{-x^2} dx$ by Hermite formula. (8)

(ii) Evaluate $\int_{1}^{2} \frac{dx}{\sqrt{3x^2+1}}$ with $n = 5$ by quadrature formula. (8)

Or

(b) (i) Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ by Gaussian three point formula. (8)

(ii) Evaluate $\int_{-1}^{1}\int_{-1}^{1} \frac{dx.dy}{xy}$ by Gaussian quadrature formula. (8)