**Reg. No. :** | | | | | | | | | | |

# Question Paper Code: 50041

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Fourth Semester

Computer Science and Engineering

15UMA421 - DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulation 2015)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. For a given statement "If it is raining, then the home team wins", the modified statement "if the home team does not win, then it is not raining" is called the _________ of the statement.

   (a) Contra-positive (b) Inverse
   (c) Converse (d) Tautology

2. For the given predicate $\exists x P(x, y)$ , the variable $x$ is called as

   (a) Quantifier (b) Free Variable
   (c) Bound variable (d) Nested Quantifier

3. The number of possible solutions of the equation $x + y + z = 15$ for $x, y, z \geq 0$ is

   (a) C(15, 3) (b) C(16, 3) (c) C(17, 2) (d) C(18, 2)

4. The minimum number of students needed to guarantee that 5 of them belong to the same subject, having the majors as physics, Chemistry, Mathematics and History is

   (a) 14 (b) 15 (c) 16 (d) 17

5. A graph that has both parallel edges and self loops is called as ________ graph.

   (a) Multi (b) Pseudo (c) Simple (d) Trivial

6. For what values of '*n*' the graph $K_n$ is Hamiltonian

(a) $n \geq 2$ (b) $n \geq 3$ (c) $n > 4$ (d) $n > 5$

7. The inverse of an element $a \in R$ such that the binary operation * is defined by $a * b = a + b + 2ab$ is

(a) $\frac{1}{2}$ (b) $\frac{a}{1+2a}$ (c) $-\frac{1}{2}$ (d) $-\frac{a}{1+2a}$

8. If $R$ is a commutative ring, then $(a \neq 0), a \in R$ is said to be zero divisor if there exists an element $(b \neq 0), b \in R$ such that

(a) $a \cdot b = 0$ (b) $a \cdot b \neq 0$ (c) $a \cdot b = 1$ (d) none of these

9. In distributive complemented lattice $a \leq b$ if and only if

(a) $a = b$ (b) $a' \oplus b = 0$ (c) $a * b' = 1$ (d) $b' \leq a'$

10. A modular lattice is distributive if and only if _______ of its sub-lattice is isomorphic to the diamond lattice $M_5$

(a) One (b) Two (c) Three (d) None of these

PART - B (5 x 2 = 10 Marks)

11. State the truth value of " If tigers have wings then the earth travels round the sun".

12. Define recurrence relation.

13. Give an example of a pair of non isomorphic graphs that have equal number of vertices, equal number of edges and the same degree sequence?

14. Define left coset of H in G.

15. Draw the Hasse diagram for $D_{30}$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Prove that $\sqrt{2}$ is irrational. (8)

(ii) Prove that $\forall x(P(x) \rightarrow Q(x)), \forall x(R(x) \rightarrow \neg Q(x)) \Rightarrow \forall x(R(x) \rightarrow \neg P(x))$. (8)

Or

(b) (i) Use rules of inference to show that the hypothesis "if it does not rain or if it is not foggy, then the Sailing race will be held and the life saving demonstration will go on" If the sailing race is held then the Trophy will be awarded" and "the trophy was not awarded" implies the conclusion "It rained". (8)

(ii) Obtain the PCNF and PDNF of $P \to (Q \wedge P) \wedge (\neg P \to (\neg Q \wedge \neg R))$. (8)

17. (a) (i) Prove by induction that for $n \geq 1$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. (8)

(ii) A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if (a) they can be any colour (b) two must be white and two red (c) they must all be the same colour. (8)

Or

(b) (i) The Using the generating function, Solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0$, $y_1 = 1$, $y_0 = 2$. (8)

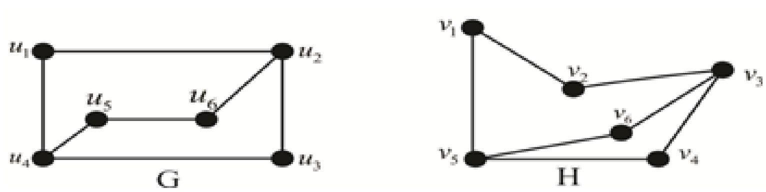
(ii) Find the number of integers between 1 and 250 both inclusive that are (a) divisible by any of the integers 2, 3, 5, 7. (b) Not divisible by any of these integers. (8)

18. (a) (i) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components $\frac{(n-k)(n-k+1)}{2}$. (8)

(ii) Prove that the number of vertices of odd degree in any graph is even. (8)

Or

(b) (i) Determine whether the graphs G and H are isomorphic. (8)




(ii) Draw the directed graph G corresponding to the adjacency matrix $A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$. Also find the In-degree and out-degree for each vertices and verify that total in-degree equal to the number of edges. (8)

19. (a) (i) State and prove Lagrange's theorem on groups. (10)

(ii) Define ring and give an example of a ring with zero divisors. (6)

Or

(b) (i) Show that $M_2$ the set of all 2 x 2 non-singular matrices over R is a group under usual multiplication. Is it Abelian. (8)

(ii) State and prove Reversal law. (8)

20. (a) (i) State and prove the distributive inequalities in a lattice. (8)

(ii) Show that De Morgan's laws $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$ hold in a complemented, distributive lattice. (8)

Or

(b) (i) In any Boolean algebra prove that

$(a' + b)(b' + c)(c' + a) = (a + b')(b + c')(c + a')$. (8)

(ii) Prove that every distributive lattice is modular. Is the converse true? Justify your claim. (8)