Reg. No. : | | | | | | | | | | |

Question Paper Code: 41051

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Fifth Semester

Computer Science and Engineering

14UMA521 - DISCRETE MATHEMATICS

(Regulation 2014)

(Common to IT Branch)

Duration: Three hours　　　　　　　　　　　　　　　　　　Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Let $P(x): x < 32$ and $Q(x): x$ is a multiple of 10 with universe of discourse as all positive integers. Then the truth value of $(\exists x)(P(x) \rightarrow Q(x))$is

   (a) True　　(b) False　　(c) 10　　(d) 20

2. $P \rightarrow Q$is equivalent to

   (a) $\urcorner Q \rightarrow P$　　(b) $Q \rightarrow P$　　(c) $P \rightarrow \urcorner Q$　　(d) $\urcorner Q \rightarrow \urcorner P$

3. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

   (a) 620　　(b) 710　　(c) 720　　(d) 610

4. The numbers of ways in which 6 boys and 4 girls be arranged in a straight line so that no two girls are together is

   (a) 10^{P_6}　　(b) 604800　　(c) 720　　(d) 17280

5. A vertex of degree one is called

   (a) Isolated vertex　　(b) Unit vertex

   (c) Pendant vertex　　(d) Proper vertex

6. The number of vertices in a regular graph of degree 4 with 10 edges is

(a) 4 (b) 10 (c) 6 (d) 5

7. The set of all real number usual multiplication is not a group, since
   (a) Multiplication is not a binary operation (b) Multiplication is not associative
   (c) Identity element does not exist (d) Zero has no inverse

8. The necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup when $a, b \in H$is
   (a) $a^{-1} * h * a \in H$ (b) $a^{-1} * b \in H$
   (c) $a^{-1} * b^{-1} \in H$ (d) $(a * b)^{-1} \in H$

9. A self-complimented, distributive lattice is called
   (a) Modular Lattice (b) Boolean Algebra
   (c) Complete Lattice (d) Self-Dual Lattice

10. The value of $(a.b)' + (a + b)'$ is
   (a) $a'.b'$ (b) $a' + b'$ (c) 0 (d) 1

PART - B (5 x 2 = 10 Marks)

11. Define quantifiers. What are its types.

12. Find the recurrence relation from $y_k = A2^k + B3^k$.

13. State any two properties of trees.

14. Draw all the spanning trees of $\boldsymbol{K_3}$.

15. Is the poset$(Z^+,/)$ a lattice?

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Obtain the principal disjunctive and principal conjunctive normal forms of $\big(P \to (Q \wedge R)\big) \wedge \big(\sim P \to (\sim Q \wedge \sim R)\big)$. (8)

(ii) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (8)

Or

(b) (i) Use the indirect method to prove that the conclusion $(\exists z)Q(z)$ follows from the premises$(\forall x)(P(x) \to Q(x))$and $(\exists y)P(y)$. (8)

(ii) Show that $(\exists x)\big(P(x) \to Q(x)\big)$ follows from the premises $\exists x\big(P(x) \wedge Q(x)\big) \to (y)\big(R(y) \to S(y)\big) and\ \exists y(R(y) \wedge \urcorner S(y))$. (8)

17. (a) (i) Solve the recurrence relation $y_{n+2} - 6y_{n+1} + 9y_n = 0,\ y_1 = 4\ and\ y_0 = 1$. (8)

(ii) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$, given that $a_0 = 2$ and $a_1 = 8$. (8)

Or

(b) (i) Show that by mathematical induction principle, $3^{2n} + 4^{n+1}$ is divisible by 5, for $n \geq 0$. (8)

(ii) Find the number of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 and 7. (8)

18. (a) (i) Find the adjacency matrix of the following graph G.

Find $A^2, A^3$and $Y = A + A^2 + A^3 + A^4$. What is your observation of entries in A^2andA^3? (8)

(ii) Prove that a connected graph is Eulerian if all vertices are of even degree. (8)

Or

(b) (i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (8)

(ii) Find the adjacency matrix of the following graph G. Find $A^2, A^3$and $Y = A + A^2 + A^3 + A^4$. What is your observation of entries in $A^2$and A^3 ? (8)

19. (a) (i) State and prove Lagrange's theorem. (8)

(ii) Define subgroup with an example. Also prove that the intersection of two subgroups of a group is also a subgroup of the group. (8)

Or

(b) (i) Prove that a non-empty subset H of a group G is a subgroup if $a, b \epsilon H \Rightarrow a * b^{-1} \epsilon H$. (8)

(ii) Let G be a group. If $a, b \in G$, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$. (8)

20. (a) (i) State and prove DeMargon's law of lattice. (8)

(ii) In a complemented, distributive lattice, prove the following:
(i) $(ab)^{'} + (a+b)^{'} = a^{'} + b'$ (ii) $ab^{'}c + ab^{'}c = b'c$ (8)

Or

(b) (i) State and prove distributive inequality of Lattice. (8)

(ii) In any Boolean algebra, show that $a'b + ab' = 0 \ iff \ a = b$. (8)