Question Paper Code: 31022

B.E. / B.Tech. DEGREE EXAMINATION, MAY 2017

Second Semester

Civil Engineering

01UMA202 - ENGINEERING MATHEMATICS - II

(Common to ALL branches)

(Regulation 2013)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

- 1. Find the particular integral of $(D^2 + 2D + 1)y = e^{-x}\cos x$.
- 2. Transform the equation $(2x+3)^2 \frac{d^2y}{dx^2} 2(2x+3)\frac{dy}{dx} 12y = 6x.$
- 3. Evaluate $\int_c (2x y)dx + (x + y)dy$ where c is the boundary of the circle $x^2 + y^2 = 1$ in the XOY plane.
- 4. Find 'a' such that $\vec{F} = (x + 3y)\vec{i} + (y 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal.
- 5. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$.
- 6. Find the Laplace transform of $e^{-t}t^2 sint$.
- 7. Show that $\frac{x}{x^2+y^2}$ is harmonic.

8. Find the invariant points of the transformation $=\frac{2z+6}{z+7}$.

9. Find the residues of
$$\frac{1-e^{2z}}{z^4}$$
 at $z = 0$.

10. Define singular point.

PART - B ($5 \times 16 = 80 \text{ Marks}$)

11. (a) (i) Solve
$$(D^2 - 3D + 2)y = 2\cos(2x + 3) + 2e^{2x}$$
. (8)

(ii) Solve
$$\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0.$$
 (8)

Or

(b) (i) Solve $(D^2 + 4)y = \cot 2x$ by method of variation of parameters. (8)

(ii) Solve
$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos[\log(1+x)].$$
 (8)

12. (a) (i) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a$, y = 0, y = b. (8)

(ii) Find
$$\nabla^2(r^n)$$
 and hence deduce $\nabla^2\left(\frac{1}{r}\right)$ where $r = |\vec{r}|$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = xz \vec{i} + 4xy \vec{j} - z^2 \vec{k}$ over the cube bounded by x = 0, x = 2, y = 0, y = 2, z = 0 and z = 2. (16)

13. (a) (i) Find the analytic function, f(z) = u + iv given $v = x^2 - y^2 + \frac{x}{x^2 + y^2}$. (8)

(ii) If
$$f(z)$$
 is a regular function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. (8)

Or

(b) (i) Find the Bilinear transformation which maps the points z = 1, i, -1 into w = i, 0, -i respectively. Also find the image of |z| < 1. (8)

(ii) Find the image of the circle |z - 3i| = 3, under the transformation $w = \frac{1}{z}$. (8)

14. (a) (i) Evaluate
$$\int_c \frac{z \, dz}{(z-1)(z-2)^2}$$
 where C is $|z-2| = \frac{1}{2}$ by using cauchy's Integral formula.
(8)

(ii) Evaluate
$$\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$$
, $a > 0, b > 0$ using contour integration. (8)

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(b) (i) Obtain Laurent's series expansion for
$$f(z) = \frac{1}{(z-1)(z-2)}$$
 in the region $|z| < 2$.
(8)

(ii) Expand
$$f(z) = \frac{1}{(z+1)(z+3)}$$
 in Laurent's series valid for the region
 $|z| > 3 \& |z| < 3.$ (8)

15. (a) (i) Using convolution theorem, find
$$L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$$
 (8)

(ii) Using Laplace transform, solve
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3t}$$
, given $y = 2$ and $\frac{dy}{dx} = 3$ when $t = 0$. (8)

Or

(b) (i) find
$$L[t^3e^{-3t}sin2t]$$
. (8)
(ii) Solve using Laplace transform $\frac{d^2y}{dx^2} + 9y = 18t$ given that $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$. (8)