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Question Paper Code : 21522

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/10177 MA 301/080100008/080210001 —
TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2008/2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the Dirichlet's conditions for Fourier series.
2. What is meant by Harmonic Analysis?
3. Find the Fourier Sine Transform of e^{-3x} .
4. If $\mathcal{F}\{f(x)\} = F(s)$, prove that $\mathcal{F}\{f(ax)\} = \frac{1}{a} \cdot F\left(\frac{s}{a}\right)$.
5. Form the PDE from $(x-a)^2 + (y-b)^2 + z^2 = r^2$.
6. Find the complete integral of $p + q = pq$.
7. In the one dimensional heat equation $u_t = c^2 \cdot u_{xx}$, what is c^2 ?
8. Write down the two dimensional heat equation both in transient and steady states.
9. Find $Z(n)$.
10. Obtain $Z^{-1}\left[\frac{z}{(z+1)(z+2)}\right]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Fourier series of x^2 in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$. (8)
- (ii) Obtain the Fourier cosine series of $f(x) = \begin{cases} kx, & 0 < x < l/2 \\ k(l-x), & l/2 < x < l \end{cases}$. (8)

Or

(b) (i) Find the complex form of Fourier series of $\cos ax$ in $(-\pi, \pi)$, where "a" is not an integer. (8)

(ii) Obtain the Fourier cosine series of $(x-1)^2$, $0 < x < 1$ and hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$. (8)

12. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence find

$$\int_0^{\infty} \frac{\sin x}{x} dx. \quad (8)$$

(ii) Verify the convolution theorem under Fourier Transform, for $f(x) = g(x) = e^{-x^2}$. (8)

Or

(b) (i) Obtain the Fourier Transform of $e^{-x^2/2}$. (8)

(ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$ using Parseval's identity. (8)

13. (a) (i) Solve : $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

(ii) Solve : $(D^2 + DD' - 6D'^2)z = y \cdot \cos x$. (8)

Or

(b) (i) Solve : $z = px + qy + \sqrt{p^2 + q^2 + 1}$. (8)

(ii) Solve : $(D^3 - 7DD'^2 - 6D'^3)z = \sin(2x + y)$. (8)

14. (a) A tightly stretched string between the fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If each of its points is given a velocity $kx(l-x)$, find the displacement $y(x, t)$ of the string.

Or

(b) An infinitely long rectangular plate is of width 10 cm. The temperature along the short edge $y = 0$ is given by

$$u = \begin{cases} 20x, & 0 < x < 5 \\ 20(10-x), & 5 < x < 10 \end{cases} \text{ If all the other edges are kept at zero}$$

temperature, find the steady state temperature at any point on it.

15. (a) (i) Find $Z(\cos n\theta)$ and hence deduce $Z\left(\cos \frac{n\pi}{2}\right)$. (8)

(ii) Using Z -transform solve : $y_{n+2} - 3y_{n+1} - 10y_n = 0$, $y_0 = 1$ and $y_1 = 0$. (8)

Or

(b) (i) State and prove the second shifting property of Z -transform. (6)

(ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$. (10)