Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 23536

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Third Semester

Civil Engineering

MA 1201/070030007 /070030005/070030004 — MATHEMATICS III

(Common to all Branches)

(Regulation 2004/2007)

(Common to B.E.(Part-Time) Second Semester, Civil Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering and Mechanical Engineering, Regulation 2005)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form a partial differential equation, by eliminating the arbitrary constants a and b from a relation $z = (x + a)(y + b)$.

2. Solve the equation $(D^3 - 3DD'^2 + 2D'^3) z = 0$.

3. State Dirichlet's conditions for a given function to expand in Fourier series.

4. Write down the complex Fourier series, stating the formula for coefficient C_n.

5. Classify the partial differential equation $u_{xx} + xu_{yy} = 0$.

6. Give the three possible solutions of the one dimensional heat flow equation.

7. Write the existence conditions of the Fourier transform.

8. State and prove the change of scale property of the Fourier transform.

9. Form the difference equation generated by $y_n = A3^n + B\,5^n$ where A and B, are arbitrary constants.

10. Find the z-transform of $(-\pi)^n$ and sketch the region of convergence.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation by eliminating the arbitrary function φ from the relation $\varphi\left(x^2 + y^2, z - xy\right) = 0$. (8)

(ii) Solve $\left(x^2 - yz\right) p + \left(y^2 - zx\right) q = \left(z^2 - xy\right)$. (8)

Or

(b) (i) Solve $\left(D^2 - DD' - 2D'^2\right) z = e^{3x+4y} + \sin(2x + 3y)$. (8)

(ii) Find the singular solution of the p.d.e. $z = px + qy + \sqrt{1 + p^2 + q^2}$. (8)

12. (a) Find the Fourier series expansion of the periodic function $f(x)$ of the period 2 defined by $f(x) = \begin{cases} 1 + x, & -1 \le x \le 0 \\ 1 - x, & 0 \le x \le 1 \end{cases}$. Deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. (16)

Or

(b) Find the first three harmonic of Fourier series of $y = f(x)$ from the following data. (16)

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
y	298	356	373	337	254	155	80	51	60	93	147	221

13. (a) A tightly stretched string of length 10 cm. with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $V_0 \sin^3\left(\frac{\pi x}{10}\right)$. Find the displacement $y(x, t)$. (16)

Or

(b) An insulated rod of length 20 cm has its ends A and B maintained at 0°C and 80°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C. Find the temperature at a distance X from A at time t. (16)

14. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. (8)

(ii) State and prove convolution theorem for Fourier transform. (8)

Or

(b) (i) Find the Fourier cosine transform of $f(x) = \begin{cases} \cos x & for\ 0 < x < a \\ 0 & for\ x > a \end{cases}$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$ and $\int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$, using Parseval's identity. (8)

15\. (a) (i) Find $z(e^{-iat})$ and hence deduce the values of $z(\cos at)$ and $z(\sin at)$. (8)

(ii) Evaluate the inverse z-transform of $\left(\frac{z^2}{(z-1)(z-3)}\right)$, by using convolution theorem. (8)

Or

(b) (i) Find the z-transform of the sequence $\frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$. (8)

(ii) Solve the difference equation $y_{n+2} + 6\ y_{n+1} + 9y_n = 2^n$ given that $y_0 = 0$ and $y_1 = 0$, by using z-transform technique. (8)
