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Question Paper Code : 53021

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Second Semester

Civil Engineering

MA 205 – MATHEMATICS – II

(Common to All Branches)

(Regulation 2007)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^{\pi} \int_0^{a\sin\theta} r\, dr\, d\theta$.

2. Find $\int_0^1 \int_0^2 \int_0^3 xdzdy\, dx$.

3. Find 'a' so that $\vec{F} = (x+z)\vec{i} + (3x+ay)\vec{j} + (x-5z)\vec{k}$ is solenoidal.

4. State Gauss divergence theorem.

5. Write the orthogonal property of an analytic function.

6. Find the fixed points of the transformation $w = \frac{z-1}{z+1}$.

7. Evaluate $\int_C \frac{z}{z+1} dz$ where C is the circle $|z| = 2$.

8. Classify the singularity of $e^{1/z}$.

9. If $L\{f(t)\}=\frac{1}{s(s+2)}$, find $\lim_{t\to\infty} f(t)$.

10. Find the inverse Laplace transform of $\frac{1}{(s+2)^3}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\iint_R (x^2+y^2)\,dx\,dy$ where R is the region enclosed by $x=0$, $y=a$; $y=x$. (8)

(ii) Find the volume of the sphere $x^2+y^2+z^2=9$ without transformation. (8)

Or

(b) (i) Evaluate $\int_0^a\int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ by changing the order of integration. (8)

(ii) Find the area of the ellipse $4x^2+9y^2=36$. (8)

12. (a) (i) Find the directional derivative of $\phi=4xz^2+x^2yz$ at (1, –2, 1) in the direction of $2\vec{i}+3\vec{j}+4\vec{k}$. (4)

(ii) Verify Stoke's theorem for $\vec{F}=(2x-y)\,\vec{i}-yz^2\vec{j}-y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is the circular boundary on the plane $z=0$. (12)

Or

(b) (i) Show that $\vec{F}=(2xy+z^3)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ is irrotational. Find its scalar potential ϕ such that $\vec{F}=\nabla\phi$. (8)

(ii) Verify Green's theorem in the plane for $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ where C is the boundary of the triangle formed by the lines $x=0, y=0, x+y=1$. (8)

13. (a) (i) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

(ii) Discuss the transformation $w=\sin z$. (8)

Or

(b) (i) Verify whether $w=(x^2-y^2-2xy)+i(x^2-y^2+2xy)$ is an analytic function of $z=x+iy$. Also express w interms of z. (8)

(ii) Show that the transformation $w=\frac{1}{z}$ maps circles and straight lines in the z-plane into circles or straight lines in the w-plane. (8)

14. (a) (i) Evaluate $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle $|z+1-i|=2$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\cos\theta}$ using method of residues. (8)

Or

(b) (i) Find the Laurent's series expansion for $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ valid in the region $1<|z+1|<3$. (8)

(ii) Using contour integration, evaluate $\int_0^{\infty} \frac{\cos x}{x^2+1}dx$. (8)

15. (a) (i) Find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ using convolution theorem. (8)

(ii) Solve, using Laplace transform, $\frac{d^2y}{dx^2}-5\frac{dy}{dt}+6y=e^{-t}$, given that $y(0)=1$ and $y'(0)=0$. (8)

Or

(b) (i) Find the Laplace transform of $f(t)=\begin{cases} t, & 0<t<1 \\ 2-t, & 1<t<2 \end{cases}$, $f(t+2)=f(t)$. (8)

(ii) Solve the integral equation $\frac{dy}{dt}+2y+\int_0^t y\,dt=\sin t, y(0)=1$ using Laplace transform method. (8)