Reg. No. :

Question Paper Code : 65147

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2013.

Third Semester

Software Engineering

EMA 003 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the partial differential equations of all planes having equal intercepts on the x and y axis.

2. Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$.

3. Write down the Fourier series if the given function $f(x)$ is an even function in the interval $(-l, l)$

4. Define root mean square value in Fourier series, for the given interval $(-l, l)$.

5. Prove that $F[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$, $a > 0$, in Fourier Transform.

6. Explain convolution of two functions in the interval $(-\infty, \infty)$ in Fourier transform.

7. If $L[f(t)] = F(s)$, then prove that $L[e^{-at} f(t)] = F(s + a)$.

8. Find the Laplace transform of $[t e^{-t} \sin t]$.

9. If $z[f(t)] = F(z)$ then prove that $z[e^{-at} f(t)] = F(z e^{aT})$.

10. Find the Z-transform of $[n a^n]$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Solve $\frac{y^2 z}{x} p + xzq = y^2$. (8)

(ii) Solve $(D^2 + 4DD' - 5D'^2)z = e^{2x-y} + \sin(x - 2y)$. (8)

Or

(b) (i) Solve $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$. (8)

(ii) Solve $(D^2 - DD')z = \cos x \cos 2y$. (8)

12. (a) (i) Find the Fourier Series for $f(x)$ if

$$f(x) = \begin{cases} -\pi & in \ -\pi < x < 0 \\ x & in \ 0 < x < \pi \end{cases}$$ (8)

(ii) Obtain the Fourier series for the period $2l$, if the function $f(x)$ is given by

$$f(x) = \begin{cases} l - x, & 0 < x \le l \\ 0, & l \le x < 2l \end{cases}.$$ (8)

Or

(b) (i) Obtain the Fourier Series at period 2π, for the given function $f(x) = x^2$ in $(-\pi, \pi)$ and hence deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots + \infty$. (8)

(ii) Expand $f(x) = x(l - x)$ over the interval $(0, l)$ as a Fourier cosine series of period l. (8)

13. (a) Find the Fourier transform of $f(x)$ if

$$f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a>0\end{cases}$$

and deduce the values of

(i) $\int_0^\infty \frac{\sin t}{t}\,dt$ and

(ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$. (16)

Or

(b) Show that the Fourier transform of

$f(x)=\begin{cases}a^2-x^2, & |x|<a\\ 0, & |x|>a>0\end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left[\frac{\sin as - as\cos as}{s^3}\right]$ and hence deduce the values of

(i) $\int_0^\infty \left[\frac{\sin t - t\cos t}{t^3}\right] dt$ and

(ii) $\int_0^\infty \left[\frac{\sin t - t\cos t}{t^3}\right]^2 dt$. (16)

14. (a) (i) Find the inverse Laplace transform of $\left[\frac{s-1}{s^2+3s+2}\right]$. (8)

(ii) Find the Laplace transform of the function

$$f(t)=\begin{cases}t, & for\ 0<t<a\\ 2a-t, & for\ a<t<2a\end{cases}$$

With the period $f(t+2a)=f(t)$. (8)

Or

(b) (i) Find the inverse Laplace transform of $\left[\frac{1}{s^2(s^2+w^2)}\right]$. (8)

(ii) Using convolution theorem, find the inverse Laplace transform of

$\left[\frac{s}{(s^2+a^2)(s^2+b^2)}\right]$. (8)

15. (a) (i) Find the Z transform of $\left[\frac{1}{(n+1)(n+2)}\right]$. (8)

(ii) Find the inverse Z – transform of $\left[\frac{4z^2-2z}{z^2-5z^2+8z-4}\right]$. (8)

Or

(b) (i) State and prove the initial and final value theorem in Z-transform. (8)

(ii) Using convolution theorem, find the inverse Z-transform of $\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$. (8)
