Reg. No. :

Question Paper Code : 65077

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2013.

Second Semester

Software Engineering

XCS 122/10677 SW 202 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to M.Sc. Information Technology and Computer Technology)

(Regulation 2003/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of $x^2 + y^2 = 1$.

2. Evaluate: $\int_0^1\int_0^2\int_0^3 xyz\,dz\,dy\,dx$.

3. Find the unit normal vector to the surface $x^2 + y^2 + z^2 = a^2$ at (1,1,1)

4. State Stoke's theorem for a plane.

5. Find the value of k if the sum of the intercept made by the plane $3x$-$6y$-$2z$-6=0 is 2

6. (2,3,4) is one end of the diameter of the sphere $x^2 + y^2 + z^2 - 2x - 2y + 4z - 1 = 0$, find the other end.

7. State C-R equations for an analytic function $f(z)$.

8. When do you say a function is harmonic? Give an example.

9. State Cauchy's Residue theorem.

10. Find the poles of $f(z) = \dfrac{z^2}{(z^2+4)}$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Change the order of integralion in $\int_0^1 \int_0^x dy\,dx$. and hence evaluate. (8)

(ii) Evaluate $\iiint_V dxdydz$, where the V is the volume of the tetrahedron whose vertices are (0,0,0), (0,1,0),(1,0,0) and (0,0,1). (8)

Or

(b) (i) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = 1$ by triple integrals. (8)

12. (a) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2dy)]$ where C is the boundary of the common area between $y = x^2$ and $y = x$. (16)

Or

(b) Verify Gauss divergence theorem for $\overline{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube bounded by the planes by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (16)

13. (a) (i) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = y-4 = \frac{z-5}{3}$ are coplanar. Find their common point. (8)

(ii) Find the equation of the spheres which has its centre at the point (-1,2,3) and touch the plane $2x - y + 2z = 6$. (8)

Or

(b) (i) Find the image of the point A(1,-2,3) in the plane $2x + y - z = 5$. (8)

(ii) Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9, x + y + z = 3$ as a great circle. (8)

14. (a) (i) Verify that the given function $u(x, y) = xy^2$ is harmonic and find its conjugate . (8)

(ii) If $f(z)$ is a regular function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|\frac{d}{dz}f(z)\right|^2$ (8)

Or

(b) (i) Show that the function $v(x, y) = e^{-x}(x\cos y + y\sin y)$ satisfies the Laplace equation. Determine the corresponding analytic function f(z). (8)

ii) If $f(z)$ is analytic, show that $f(z)$ is a constant if real part of $f(z)$ is a constant. (8)

15. (a) (i) Evaluate $\oint_C \frac{z^2+1}{z^2-1}\,dz$ where C is the circle of unit radius and centre at $z = i$. (6)

(ii) Show that $\int_0^{2\pi} \frac{d\theta}{5+2\cos\theta} = \frac{2\pi}{\sqrt{21}}$ by contour integration. (10)

Or

(b) (i) Obtain the Laurent's series to represent the function $\frac{7z-2}{z(z-2)(z+1)}$, in the region $1 < |z+1| < 3$. (8)

(ii) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+9)(x^2+4)}\,dx = \frac{\pi}{10}$ by contour integration method. (8)
