LIB
26/6/13 Fn

Reg. No. :

Question Paper Code : 65254

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2013.

Second Semester

Software Engineering

EMA 002 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Regulation 2010)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_{-1}^{2}\int_{x}^{x+2} dydx$.

2. Change the order of integration in $\int_{0}^{a}\int_{y}^{a} \frac{x+y}{x^2+y^2}\,dxdy$.

3. Define divergence of a vector $\overline{F}$.

4. State Gauss divergence theorem.

5. Write the equation of the plane which passes through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

6. What is the length of the perpendicular from the origin on the plane $ax + by + cz + d = 0$?

7. If $f(z)$ is analytic and if $f'(z) = 0$ everywhere, show that $f(z)$ is a constant.

8. Define a Harmonic function.

9. State Cauchy's integral theorem.

10. Evaluate $\int_C \frac{dz}{z-a}$, where C is a simple closed curve and $z = a$ is a point outside C.

PART B — (5 × 16 = 80 marks)

11. (a) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by triple integrals.

Or

(b) Change the order of integration and hence find the value of $I=\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx$.

12. (a) Verify Stoke's theorem for $\vec{F}=(y-z)\hat{i}+yz\hat{j}+xz\hat{k}$, where S is the open surface bounded by the planes $x=0$, $x=1, y=0$, $y=1$ and $z=1$ above the xoy plane

Or

(b) Verify Gauss divergence theorem for, $\overline{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken over the rectangular parallelo piped enclosed by $x=0$, $x=a$, $y=0$, $y=b$ $z=0$ and $z=c$.

13. (a) Find the length of the shortest distance between the lines $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z}{4}$ and $2x+3y-5z-6=0=3x-2y-z+3$.

Or

(b) Show that the plane $2x+2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$ and find also the point of contact.

14. (a) Show that the function $u(x,y)=3x^2y+2x^2-y^3-2y^2$ is harmonic. Find the conjugate harmonic function v and express $u+iv$ as an analytic function of z.

Or

(b) If $f(z)$ is a regular function of z, prove that $\left[\frac{\partial^2}{\partial x}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$.

15. (a) Find the Laurent's series expansion for the function $f(z)=\frac{7z-2}{(z-1)(z-4)}$ in the region

(i) $|z-1|<1$

(ii) $1<|z-1|<2$.

Or

(b) Show that $\int_0^{\infty}\frac{\cos mx}{a^2+x^2}=\frac{\pi}{2a}e^{-ma}$, $m>0$.
