Reg. No. :

Question Paper Code : 65193

5 year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2013.

Fourth Semester

Software Engineering

EMA 005 — DISCRETE MATHEMATICS

(Regulation 2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define negation and conjunction.
2. Obtain the principal conjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R)$
3. Define equivalence relations.
4. Define the composition of two functions.
5. Define subgroup.
6. What is meant by group homomorphism?
7. Define ring.
8. Define polynomial ring.
9. State the properties of lattices.
10. Define a Boolean algebra with an example.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Construct the truth table for $(P \vee Q) V \neg P$. (8)

(ii) Obtain a conjunctive normal for of the following formula $\neg(P \vee Q) \rightleftharpoons (P \wedge Q)$. (8)

Or

(b) (i) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$. (8)

(ii) Show that $(x)(p(x) \rightarrow Q(x) \wedge x(Q(x) \rightarrow R(x)) \Rightarrow (x)\,(P(x) \rightarrow R(x))$. (8)

12. (a) (i) Let $X = \{1, 2, ...7\}$ and $R = \{\langle x, y \rangle / x - y$ is divisible by $3\}$ show that R is an equivalence relation. Draw the graph of R. (8)

(ii) Let R and S be two relations on a set of positive integers I :

$R = \{\langle x, 2x \rangle / x \in I\}$, $S = \{\langle x, 7x \rangle / x \in I\}$ find $R \circ S$, $R \circ R$, $R \circ R \circ R$ and $R \circ S \circ R$. (8)

Or

(b) (i) If X and Y are finite sets, find a necessary condition for the existence of one-to-one mappings from X to Y. (8)

(ii) Let F_x be the set of all one-to-one onto mappings from X on to X, where $X = \{1, 2, 3\}$. Find all the elements of F_x and find the inverse of each element. (8)

13. (a) (i) Show that in a group $\langle G, *\rangle$, if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$, then $\langle G, *\rangle$ must be abelian. (8)

(ii) Show that the set of all elements a of a group $\langle G, *\rangle$ such that $a * x = x * a$ for every $x \in G$ is a subgroup of G. (8)

Or

(b) (i) Show that if $\langle G, *\rangle$ is a cyclic group, then every subgroup of $\langle G, *\rangle$ must be cyclic. (8)

(ii) Find the left cosets of $\{[0],[3]\}$ in the group $\langle z_6 +_6 \rangle$. (8)

14. (a) (i) Explain ring homomorphism with an example. (8)

(ii) Show that the ring of even integers is a subring of the rings of integers. (8)

Or

(b) (i) Show that $\langle i, \oplus, \odot \rangle$ is a commutative ring with identity, where the operations $\oplus$ and $\odot$ are defined, for any $a, b \in I$, as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$. (8)

(ii) For any integer m, show that $\{xm / x \in I\}$ is a subring of the ring of integers. (8)

15. (a) (i) Show that the operations of meet and join on a lattice are commutative, associative and idempotent. (8)

(ii) Show that in a lattice if $a \leq b$ and $c \leq d$, then $a * c \leq b * d$. (8)

Or

(b) (i) Find the complements of every element of the lattice $\langle S_n, D \rangle$ for $n = 75$. (8)

(ii) Simplify the following Boolean expression

(1) $(a * b)' \oplus (a \oplus b)'$ and (4)

(2) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$. (4)
