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Question Paper Code : 65159

5 years M.Sc. DEGREE EXAMINATION, MAY/JUNE 2013.

First Semester

Software Engineering

EMA 001— TRIGNOMETRY, ALGEBRA AND CALCULUS

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

PART A — (10 × 2 = 20 marks)

1. Expand $Cos4\theta$ in a series of powers of $Cos\theta$.

2. Prove that $\log\left\{\frac{(1+i)(2+i)}{(3+i)}\right\}$ is imaginary.

3. State Cayley-Hamilton theorem.

4. Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$.

5. Find the Taylor Series expansion of x^y about the point (1,1) upto the first degree terms.

6. If $u - Sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

7. Evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)^{3/2}}$.

8. Evaluate $\int_0^\pi \sin^8 x\, dx$.

9. Find the particular integral of $(D^2+4D+4)y = xe^{-2x}$.

10. Solve the equation $x^2y^{11} - xy^1 + y = 0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $\frac{Sin7\theta}{Sin\theta} = 7 - 56Sin^2\theta + 112Sin^4\theta - 64Sin^6\theta$. (8)

(ii) Separate the real and imaginary parts of $\tan(x+iy)$. (8)

Or

(b) (i) Expand $Cos^7\theta$ in a series of cosines of multiples of θ. (6)

(ii) If $x + iy = C\cos(A - iB)$. Prove that

(1) $\frac{x^2}{C^2 \cos h^2 B} + \frac{y^2}{C^2 \sin h^2 B} = 1$. and

(2) $\frac{x^2}{C^2 \cos^2 A} + \frac{y^2}{C^2 \sin^2 A} = 1$. (10)

12. (a) (i) Investigate for what values of a, b the simultaneous equations.

$x + y + 2z = 2,\ 2x - y + 3z = 2,$
$5x - y + az = b$ have

(1) no solution

(2) a unique solution

(3) an infinite number of solutions. (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the matrix. (8)

$$\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

Or

(b) (i) Diagonalise the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ an d hence find A^6. (8)

(ii) Find the eigenvalues and the eigenvectors of $\begin{bmatrix} 3 & -1 & 3 \\ 9 & -1 & 9 \\ 7 & -1 & 7 \end{bmatrix}$. (8)

13. (a) (i) If $z = f(x, y) and dx = r\cos\theta, y = r\sin\theta$.

Prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{r\theta}\right)^2$. (8)

(ii) Find the relative extrema of $x^3 + y^3 - 3x - 12y + 20$. (8)

Or

(b) (i) Find the Taylor expansion of $e^x Siny$ near the point $\left(-1, \frac{\pi}{4}\right)$ upto the third degree terms. (8)

(ii) If $x = u + v + w, xy = v + w, xyz = w$,

Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = x^2 y$. (8)

14. (a) (i) If $u_n = \int_0^a x^n e^{-x} dx$, prove that

$u_n - (n+a)u_{n-1} + a(n-1)u_{n-2} = 0$. (6)

(ii) Find the area of the loop of the curve $y^2 = x^2(4 - x^2)$ between $x = 0$ and $x = 2$. (10)

Or

(b) (i) Find the length of the loop of the curve $9ay^2 = x(x - 3a)^2$. (10)

(ii) Prove that $\int_0^{\frac{\pi}{2}} \log(Sinx)dx = \frac{-\pi}{2}\log 2$. (6)

15. (a) (i) Solve : $(D2 - 4D + 3)y = Sin3x\cos 2x + e^x(x^2 + 2)$. (8)

(ii) Solve : $\frac{dx}{dt} + 2y = 5e^t$

$\frac{dx}{dt} - 2x = 5e^t$. (8)

Or

(b) (i) Solve : $(5+2x)^2 y^{11} - 6(5+2x)y^1 + 8y = 6x$. (8)

(ii) Solve : $(D^2 + 4)y = x\cos x$. (8)
