Reg. No. :

Question Paper Code : 65043

5 Year M.Sc. DEGREE EXAMINATION, MAY/JUNE 2013.

Third Semester

Computer Technology

XCS 231/10677 SW 301 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 Year M.Sc. Information Technology and Software Engineering)

(Regulation 2003/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the complete solution of $p^2 + q^2 = 1$.

2. Form a partial differential equation by eliminating the arbitrary φ function from $z = \varphi\left(\frac{x}{y}\right)$.

3. Find the constant term a_0 in the Fourier series expansion of $f(x) = x + x^2, \text{in } (-\pi, \pi)$.

4. Find the RMS value of $f(x) = 1$ in $0 < x < \pi$.

5. Find the Fourier transform of $f(x) = e^{ikx}, a < x < b$.

6. Find the Fourier sine transform of $f(x) = e^{-x}$.

7. Find $L\left(e^{-2t} \cos t\right)$.

8. Verify final value theorem of Laplace transform for $f(t) = 1 - e^{-at}$.

9. Find $Z\left((-1)^n\right)$.

10. Find $Z\left(\cos\left(\frac{n\pi}{2}\right)\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation: $z = px + qy + c\sqrt{1 + p^2 + q^2}$. (8)

(ii) Solve the equation: $y^2 p - xyq = x(z - 2y)$. (8)

Or

(b) (i) Solve: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

(ii) Solve: $(D^2 - 2DD' + 2D')z = (x^2 y^2 e^{x+y})$. (8)

12. (a) (i) Find the half – range consine series of $f(x) = x(\pi - x)$ in $(0, \pi)$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots\ldots$ (8)

(ii) Find the Fourier series expansion of $f(x) = e^{-x}$ in $(-\pi, \pi)$. Hence obtain a series for $\frac{\pi}{\sin h\pi}$. (8)

Or

(b) (i) Find the Fourier series expansion of $f(x)$ given by $f(x) = \begin{cases} 1, \text{ in } 0 < x < 1 \\ 2, \text{ in } 1 < x < 3 \end{cases}$. (8)

(ii) Find the half–range sine series of $f(x) = x$ in $(0,1)$. Hence find the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots$ (8)

13. (a) (i) Find the Fourier cosine transform of $f(x)$ defined as $f(x) = \begin{cases} x, \text{ for } 0 < x < 1 \\ 2 - x, \text{ for } 1 < x < 2 \\ 0, \text{ for } x > 2 \end{cases}$. (6)

(ii) Find the Fourier sine transform of e^{-ax} $(a > 0)$. Hence find $F_s\{xe^{-ax}\}$ and $F_s\left\{\frac{e^{-ax}}{x}\right\}$. Deduce the value of $\int_0^\infty \frac{\sin sx}{x} dx$. (10)

Or

(b) Find the Fourier transform of $f(x)=\begin{cases}1-x^2 \text{ for } |x|<1\\ 0, \text{for } |x|>1\end{cases}$. Hence find

(i) $\int_0^\infty \frac{\sin x - x\cos x}{x^3}\cos\left(\frac{x}{2}\right)dx$ and

(ii) $\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$. (16)

14. (a) (i) Find the inverse Laplace transform of $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$. (8)

(ii) Using convolution theorem, find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$. (8)

Or

(b) (i) Solve: $f(t)=4t-3\int_0^t f(u)\sin(t-u)\,du$. (8)

(ii) Find the Laplace transform of the periodic function $f(t)=\begin{cases}E, \text{ in } 0\le t\le \frac{T}{2}\\ -E, \text{ in } \frac{T}{2}\le t\le T\end{cases}$ given that $f(t+T)=f(t)$. (8)

15. (a) (i) Find $Z(r^n \cos n\theta)$, $Z(r^n \sin n\theta)$, $Z(\cos n\theta)$ and $Z(\sin n\theta.)$ (10)

(ii) Find the Z – transform of $f(n)=\frac{2n+3}{(n+1)(n+2)}$. (6)

Or

(b) (i) Solve the equation $f(n)+3f(n-1)-4f(n-2)=0, n\ge 2$, given that $f(0)=3$ and $f(1)=-2$. (10)

(ii) Find the inverse Z – transform of $\frac{z}{z^2+7z+10}$. (6)

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