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M.E./M.Tech. DEGREE EXAMINATION, JUNE/JULY 2013.

First Semester

Communication Systems

MA 9218/MA 909 — APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Common to M.E., Computer and Communication, M.E. Digital Signal Processing, and M.Tech. Information and Communication Technology)

(Regulation 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

2. State the orthogonality property of Bessel functions.

3. State the singular value decomposition theorem.

4. Discuss the nature of the matrix $A = \begin{bmatrix} 4 & 10 & 14 \\ 10 & 41 & 59 \\ 14 & 59 & 94 \end{bmatrix}$.

5. Find the mean of the Geometric distribution.

6. If X is a uniform random variable in $[-2, 2]$ find the pdf of X and $Var(X)$.

7. Prove that the correlation coefficient always lies between –1 and 1.

8. If the joint pdf of the random variables (x, y) is given by $f(x, y) = kxye^{-(x^2+y^2)}$, $x > 0$, $y > 0$ find the value of K.

9. Define Markovian queueing models.

10. Give an example of a queue with infinite number of servers.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^n J_{n+1}(x)$. (8)

(ii) Prove that $\frac{x}{2} = J_1(x) + 3J_3(x) + 5J_5(x) + \ldots\ldots\infty$. (8)

Or

(b) (i) Prove that $J_0^2(x) + 2\left[J_1^{\,2}(x) + J_2^{\,2}(x) + J_3^{\,2}(x) + \ldots\ldots\right] = 1$. (8)

(ii) Express $J_{3/2}(x)$ and $J_{-5/2}(x)$ in closed form. (8)

12. (a) (i) Find the Cholesky factorization of this real, symmetric and positive definite matrix $A = \begin{bmatrix} 4 & 10 & 14 \\ 10 & 41 & 59 \\ 14 & 59 & 94 \end{bmatrix}$. (8)

(ii) Find the QR – factorization of this matrix $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 2 \\ 1 & 0 \end{pmatrix}$. (8)

Or

(b) (i) Find the singular value decomposition of $A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$. (8)

(ii) Find the least – squares solution of the system $x_1 + 4x_2 = 1$, $2x_1 + 5x_2 = 1$, $3x_1 + 6x_2 = -2$. (8)

13. (a) (i) Derive the MGF of Poisson distribution and hence derive its mean and variance. (8)

(ii) The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70 with a SD of Rs. 5. Estimate the number of workers whose weekly wages will be

(1) between Rs. 69 and Rs. 72,

(2) Less than Rs. 69

(3) More than Rs. 72. (8)

Or

(b) (i) A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are

(1) Exactly three defectives (8)

(2) not more than 3 defectives.

(ii) Let X be a continuous random variable with pdf

$$f(x)=\begin{cases}\frac{x}{2} & in \quad 1<x<5 \\ 0 & elsewhere\end{cases}$$

find the probability density function of $Y=2X-3$. (8)

14. (a) (i) The two dimensional random variable (X,Y) has the joint density function

$f(x,y)=\frac{x+2y}{27}$, $x=0,1,2;$ $y=0,1,2.$ Find the marginal distributions. Also find the conditional distribution of Y for $X=x$ and the conditional distribution of X given $Y=1$. (8)

(ii) Find the co – efficient of correlation between industrial production and export using the following data : (8)

Production (X)	55	56	58	59	60	60	62
Export (Y)	35	38	37	39	44	43	44

Or

(b) (i) If the joint pdf of X and Y is given by

$$f(x,y)=\begin{cases}\frac{1}{8}(6-x-y), & 0<x<2, \quad 2<y<4 \\ 0, & elsewhere\end{cases}$$

Find

(1) $P(X<1 \cap Y>3)$

(2) $P(X<1/Y>3)$. (8)

(ii) If X and Y are independent random variables each normally following $N(0,2)$, find the probability density function of $Z=2X+3Y$. (8)

15. (a) (i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 3 per minute, find the probability that the interval between 2 consecutive arrivals is

(1) more than 1 minute

(2) between 1 minute and 2 minutes

(3) 4 minutes or less. (8)

(ii) A self – service stores employs one cashier at its counter. Nine customers arrive on an average of 5 minutes, while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate find

(1) average number of customers in the system

(2) average number of customers in the queue

(3) average time customer waits before being served

(4) average time a customer spends in the system. (8)

Or

(b) (i) Customers arrive at a watch repair shop according to Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes. Find the average number of customers, the average waiting time a customer spends in the shop and the average time a customer spends in the queue. (8)

(ii) A supermarket has two servers servicing at counters. The customer arrive in a Poisson fashion at the rate of 10 per hour. The service time for each customer is exponential with 4 mean minutes. Find the probability that a customer has to wait for the service, average queue length, and the average time spent by a customer in the queue. (8)