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Reg. No. :

Question Paper Code : 71431

M.E. DEGREE EXAMINATION, JUNE/JULY 2013.

First Semester

Structural Engineering

MA 9212/MA 9321/MA 903/UMA 9103/10211 AM 101 — APPLIED MATHEMATICS

(Common to M.E. Soil Mechanics and Foundation Engineering)

(Regulation 2009/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — $(10 \times 2 = 20$ marks)

1. Prove that Laplace Transform is linear.
2. State modulation property on Fourier Transform.
3. State any two properties of harmonic functions.
4. Write down the two dimensional Laplace equation in polar co-ordinates.
5. Define functional and give an example.
6. State the Brachistochrone problem.
7. Define Eigen value of a matrix.
8. What is the use of Faddeev-Leverrier method?
9. What is the physical meaning of $\int_a^b f(x)\,dx$?
10. What is the order of error in Trapezoidal rule of integration?

PART B — (5 × 16 = 80 marks)

11. (a) A string is stretched and fixed between two points $(0,0)$ and $(l,0)$. Motion is initiated by displacing the string in the form $u = \lambda \sin\left(\frac{\pi x}{l}\right)$ and released from rest at time t = 0. Find the displacement of any point on the string at any time 't'. (16)

Or

(b) Solve the heat conduction problem using Fourier transform described by

$PDE : k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 < x < \infty, t > 0$

B.C's : $u(0,t) = u_0, t \geq 0$

I.C's : $u(x,0) = 0, 0 < x < \infty$

u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. (16)

12. (a) Solve the following Neumann problem described by :

PDE : $u_{xx} + u_{yy} = 0, -\infty < x < \infty, y > 0$

BC's : $u_y(x,0) = f(x), -\infty < x < \infty$,

u is bounded as $y \to \infty$,

u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$.

Or

(b) Using the method of integral transform, solve the following potential problem in the semi-infinite strip described by $u_{xx} + u_{yy} = 0, 0 < x < \infty, 0 < y < a$ subject to $u(x,0) = f(x)$, $u(x,a) = 0, u(x,y) = 0$, in $0 < y < a, 0 < x < \infty$ and $\frac{\partial u}{\partial x}$ tends to zero as $x \to \infty$. (16)

13. (a) (i) Find the extremal of the functional $\int_0^{\pi} \left(4y\cos x + y'^2 - y^2\right)dx$ with $y(0) = 0, y(\pi) = 0$. (8)

(ii) Find the extremal of the functional $I(y,z) = \int_0^1 \left(y'^2 + z'^2 + 2y\right)dx$ with $y(0) = 1, y(1) = 3/2, z(0) = 0$ and $z(1) = 1$. (8)

Or

(b) (i) Find the shortest distance between the circle $x^2 + y^2 = 1$ and the straight line $x + y = 1$ using calculus to variations. (8)

(ii) Find the extremal of the functional $V = [y(x)] = \int_{x_0}^{x_1} \left(2xy + y'''^2\right) dx$. (8)

14. (a) Using power method, find the dominant eigen value and the corresponding eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Find also the least eigen value and hence the 3rd eigen value. (16)

Or

(b) Using Rayleigh-Ritz method solve the equation $y'' + x = 0$ with $y(0) = 0 = y(1)$ and compare your result with actual solution. (16)

15. (a) (i) Evaluate $\int_{-2}^{2} e^{-x/2}\, dx$ by Gaussian two point formula. (8)

(ii) Use Gaussian three point formula and evaluate $\int_{-1}^{1} \left(3x^2 + 5x^4\right) dx$. (8)

Or

(b) Using numerical method evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x + y)\, dx dy$ and compare your result by actual integration. (16)

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