Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 53020

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

First Semester

Civil Engineering

MA 105 – MATHEMATICS – I

(Common to all Branches)

(Regulation 2007)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.

2. Explain, how to obtain the power of a square matrix using the principle of diagonalisation?

3. Write the equation of the cone whose vertex is the origin and base the circle $x = a, y^2 + z^2 = b^2$.

4. Find the angle between the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

5. Determine the curvature of the straight line $y = ax + b$ at an arbitrary point (x, y).

6. Find the envelope of the family of straight lines $y = mx + am^2$, m being the parameter.

7. If $f(x,y) = x^4 y^2 \sin^{-1}\left(\frac{y}{x}\right)$, then find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

8. If $x\sin(x-y) - (x+y) = 0$; find $\frac{dy}{dx}$.

9. Solve $(D^2+1)^2 y = 0$, where $D = \frac{d}{dx}$.

10. Find the particular integral of $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1)$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) Verify Cayley–Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

Hence compute A^{-1}.

Or

(b) Reduce $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by an orthogonal transformation.

12. (a) (i) Find the shortest distance and its equations between the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$. (8)

(ii) Find the equations to the lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. (8)

Or

(b) (i) Find the equation of the sphere which passes through the points (1,0,0), (0,1,0), (0,0,1) and has its radius as small as possible. (8)

(ii) Show that the following lines are coplanar :

$$\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2} \text{ and } 3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4.$$

13. (a) (i) Find the equation of the evolute of an ellipse represented by the parametric equations $x = a\cos t, y = b\sin t$. (8)

(ii) Find the circle of curvature of the curve $x^3 + y^3 = 3xy$ at $(3/2, 3/2)$ on it. (8)

Or

(b) (i) Find the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a, 0)$. (8)

(ii) Find the envelope of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a + b = c$. (8)

14. (a) (i) Expand $e^x \log(1+y)$ in powers of x and y upto third degree terms. (8)

(ii) Examine the function $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ for maxima and minima. (8)

Or

(b) (i) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (10)

(ii) If $\varphi(x - az, cy - bz) = 0$, then show that $a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = C$. (6)

15. (a) (i) Solve $(x^2D^2 - xD + 4)y = \cos(\log x) + x\sin(\log x)$. (8)

(ii) Solve $(D^3 - D^2 - 6D)y = x^2 + 1$. (8)

Or

(b) (i) Solve the system :

$$\frac{dx}{dt} = 2y,\ \frac{dy}{dt} = 2z,\ \frac{dz}{dt} = 2x.$$ (10)

(ii) Derive the governing equation of an L–C–R circuit. (6)