

7. Let S be a non empty set and * be a binary operation on S. The algebraic system (S, *) is called a ___ if the operation * is associative. CO4- R
- (a) Group (b) Semigroup (c) Monoid (d) Abelian
8. A subgroup (H,*) of (G,*) is called a _____ if for any $a \in G$, $aH=Ha$ CO4- R
- (a) Group (b) Normal subgroup (c) Abelian group (d) Cyclic group
9. The complemented, distributive lattice is a CO5- R
- (a) Boolean algebra (b) Distributive (c) Lattice (d) Sub lattice
10. $a \bullet b + a \bullet b' =$ CO5- R
- (a) a (b) b (c) a' (d) b'

PART – B (5 x 2= 10 Marks)

11. Give the truth value of $T \leftrightarrow T \wedge F$. CO1- R
12. State the pigeonhole principle. CO2- R
13. How many edges are there in a graph with 10 vertices each of degree 5? CO3- R
14. Show that every cyclic group is abelian. CO4- R
15. Show that the absorption laws are valid in a Boolean Algebra. CO5- R

PART – C (5 x 16= 80 Marks)

16. (a) (i) Obtain the PCNF and PDNF of $(P \wedge Q) \vee (\neg P \wedge R)$ CO1- Ana (8)
- (ii) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M, \neg M$ CO1- Ana (8)
- Or
- (b) (i) Show that CO1- Ana (8)
- $$(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$$
- (ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ without using truth table. CO1- Ana (8)

17. (a) (i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2, 3, 5 & 7. CO2- App (8)

(ii) Using Mathematical induction show that $\sum_{r=0}^n 3^r = \frac{3^{n+1} - 1}{2}$ CO2- App (8)

Or

(b) (i) Show that the recurrence relation $a_n - 7a_{n-1} + 6a_{n-2} = 0$ for $n \geq 2$ with initial conditions $a_0 = 8$ and $a_1 = 6$, using generating function. CO2- App (8)

(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if
 (a) they can be male or female,
 (b) two must be men and two women,
 (c) they all are of the same sex. CO2- App (8)

18. (a) (i) Prove that in a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. CO3- Ana (8)

(ii) Prove that a connected graph G is an euler graph if and only if all vertices of G are of even degree. CO3- Ana (8)

Or

(b) (i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic? CO3- Ana (10)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) State and prove handshaking theorem. CO3- Ana (6)

19. (a) (i) Show that $(Q^+, *)$ is an abelian group where $*$ is defined by $a * b = \frac{ab}{2}$, $\forall a, b \in Q^+$ CO4- Ana (8)

(ii) Prove that the kernel of a homomorphism is a normal subgroup of G . CO4- Ana (8)

Or

- (b) (i) Prove that the intersection of two normal subgroups of a group G is again a normal subgroup of G . CO4- Ana (8)
- (ii) Let G be a finite group and H be a subgroup of G . Then prove that the order of H divides order of G . CO4- Ana (8)
20. (a) (i) Let $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and let the relation R be divisor on D_{30} . Find (a) all the lower bounds of 10 and 15. CO5- App (8)
- (b) the glb of 10 and 15.
- (c) all upper bounds of 10 and 15.
- (d) the lub of 10 and 15.
- (e) draw the Hasse diagram.
- (ii) Prove that in a Boolean algebra $(a \vee b)' = a' \wedge b'$ CO5- App (8)
- Or
- (b) (i) Show that every chain is a distributive lattice. CO5- App (8)
- (ii) Show that every chain is a modular. CO5- App (8)