A

Reg. No: | | | | | | | | | | |

Question Paper Code: 51002

B.E./B.Tech. DEGREE EXAMINATION, APRIL 2019

First Semester

Civil Engineering

15UMA102 – ENGINEERING MATHEMATICS-I

(Common to All Branches)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Evaluate $\lim_{x\to 3} \frac{x^2-9}{x-3}$ CO1- R

(a) 0 (b) 4 (c) 1 (d) 6

2. Suppose $(x) = \begin{cases} \frac{x^2-x}{2x} & if\ x \neq 0 \\ k & if\ x = 0 \end{cases}$. If $f(x)$ is continuous at x=0, then the value of 'k' is CO1- R

(a) -1 (b)1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

3. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then by Euler's theorem the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is CO2- R

(a) 2u (b) u (c) 3u (d) 0

4. If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$then $\frac{\partial(u,v)}{\partial(x,y)}$ is CO2- R

(a) -3 (b) 3 (c) 0 (d) 1

5. Value of $\Gamma\left(\frac{1}{2}\right)$ is CO3- R

(a) $\frac{\pi}{2}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\pi}$ (d) π

6. Evaluation of $\int x^{-5}\,dx$ is CO3- R

(a) $\frac{x^4}{4}+C$ (b) $-\frac{1}{4x^4}+C$ (c) $\frac{x}{2}+C$ (d) None of these

7. Value of the double integral $\int_0^1\int_0^y dydx$ is CO4- R

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

8. The value of $\int_0^{\frac{\pi}{2}}\int_0^{sin\theta} rdrd\theta$is CO4- R

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{8}$ (c) π (d) $\frac{\pi}{3}$

9. The eigen values of $\begin{bmatrix}5 & 6 & 17\\ 0 & -9 & 23\\ 0 & 0 & 37\end{bmatrix}$ are CO5 R

(a) 5, -9, 23 (b) 6, -9, 37 (c) 17, 6, 5 (d) 5, -9, 37

10. Sum of the eigen values of the matrix $\begin{pmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{pmatrix}$is CO5 R

(a) 3 (b) 5 (c) 6 (d) 0

PART – B (5 x 2= 10Marks)

11. State Leibnitz's theorem to find nth derivative of product of two functions. CO1- App

12. FIf $x = rcos\theta, y = rsin\theta, then\ find\ J(u,v)$. CO2- App

13. Evaluate $\int x^2\sqrt{x^3+1}\,dx$. CO3- App

14. Evaluate $\int_0^1\int_1^2 x(x+y)dydx$ CO4- App

15. If the sum of two eigenvalues and trace(=sum of diagonal) of a 3X3 matrix A are equal, find the value of $|A|$. CO5- App

PART – C (5 x 16= 80Marks)

16. (a) (i) Find CO1 -App (4)

$\lim_{x\to 1}\frac{\sqrt{5x-4}-\sqrt{x}}{x-1}$

(ii) Evaluate $\lim_{n\to\infty}\frac{5^{n+1}+7^{n+1}}{5^n-7^n}$ CO1 -App (4)

(iii) Expand $\tan x$ upto the term containing x^5, using Maclaurin's series. CO1 -App (8)

Or

(b) (i).Find y_n if $y = x^{n-1}logx$ CO1 -App (8)

(ii)If $y = e^{ax}\sin(bx)$ then prove that CO1 -App (8)

$y_2 - 2ay_1 + (a^2 + b^2)y = 0$

17. (a) (i) Verify Euler's theorem for the function $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ CO2 -App (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 , if $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_3x_1}{x_2}$ and $y_3 = \frac{x_1x_2}{x_3}$. CO2 -App (8)

Or

(b) (i) Examine $f(x, y) = x^3 + y^3 - 12x - 3y + 20$ for its extreme values. CO2 -App (8)

(ii) The temperature T at any point (x, y, z) in a space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. CO2 -App (8)

18. (a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ CO3 -Ana (16)

Or

(b) (i) Evaluate $\int \frac{x^2}{(x-1)^3\,(x-2)}\,dx$ CO3 -E (8)

(ii) Evaluate $\iint [xy[1 - x - y]^{\frac{1}{2}}\,dx\,dy$, over the lines enclosed by the lines $x = 0, y = 0\ and\ x + y = 1$in the positive quadrant. CO3 -E (8)

19. (a) (i) Change the order of integration and hence evaluate $\int_0^a \int_y^a \frac{x}{\sqrt{x^2+y^2}}$ dx dy CO4 -E (8)

(ii) Evaluate $\iiint_S\ xyz\,dxdydz$, where $S = \{(x, y, z): x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0, z \geq 0\}$ CO4 -E (8)

Or

(b) (i) Find the area of the region outside the inner circle $r = 2\,cos\theta$ and inside the outer circle $r = 4cos\theta$ by double integration. CO4 -App (8)

(ii) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the ordinate axis. CO4 -App (8)

20. (a) (i) Find the eigenvalues and eigenvectors of the matrix CO5 -App (8)

$$A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$$

(ii) Using Cayley Hamilton theorem, find the value of the matrix CO5 -App (8)

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 - 8A^2 + 2A - I$,

$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.

Or

(b) Reduce the Quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation and hence show that it is positive semi definite. Give also a non-zero set of values which will make the quadratic form zero. CO5-Ana (16)