Reg. No. : | | | | | | | | | | |

Question Paper Code: 45021

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Fifth Semester

Computer Science and Engineering

14UMA521 - DISCRETE MATHEMATICS

(Regulation 2014)

(Common to IT Branch)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $(P \rightarrow Q) \rightarrow Q$ is equivalent to

   (a) Q   (b) P   (c) $P \vee Q$ (d) $P \wedge Q$

2. Let $P(x): x < 32$ and $Q(x): x$ is a multiple of 10 with universe of discourse as all positive integers. Then the truth value of $(\exists x)(P(x) \rightarrow Q(x))$is

   (a) True   (b) False   (c) 10   (d) 20

3. The solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$ is

   (a) $a_n = 2^n + (-1)^n$ (b) $a_n = (1+n)(3)^n$

   (c) $a_n = 3.2^n - (-1)^n$ (d) $\left(1+\frac{n}{3}\right)(3)^n$

4. The numbers of ways in which 6 boys and 4 girls be arranged in a straight line so that no two girls are together is

   (a) 10^{P_6} (b) 604800   (c) 720   (d) 17280

5. A vertex of degree one is called

   (a) Isolated vertex   (b) Unit vertex   (c) Pendant vertex   (d) Proper vertex

6. The number of vertices in a regular graph of degree 4 with 10 edges is
   (a) 4 (b) 10 (c) 6 (d) 5

7. The set of all real number usual multiplication is not a group, since
   (a) Multiplication is not a binary operation (b) Multiplication is not associative
   (c) Identity element does not exist (d) Zero has no inverse

8. The necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup when $a, b \in H$is
   (a) $a^{-1} * h * a \in H$ (b) $a^{-1} * b \in H$
   (c) $a^{-1} * b^{-1} \in H$ (d) $(a * b)^{-1} \in H$

9. The value of $(a.b)' + (a+b)'$ is
   (a) $a'.b'$ (b) $a' + b'$ (c) 0 (d) 1

10. A Lattice $(L, \wedge, \vee)$ is said to be modular if for $a \leq c$, then
    (a) $a \vee b = b \vee c$ (b) $a \vee (b \vee c) = a \wedge (b \wedge c)$
    (c) $a \wedge (b \wedge c) = a \vee (b \wedge c)$ (d) $a \vee (b \wedge c) = (a \vee b) \wedge c$

PART - B (5 x 2 = 10 Marks)

11. Using truth table, show that $P \vee \urcorner (P \wedge Q)$ is tautology.

12. Find the recurrence relation from $y_k = A2^k + B3^k$.

13. Give an example of a graph which is both Eulerian and Hamiltonian.

14. Draw all the spanning trees of $\boldsymbol{K_3}$.

15. Is the poset$(Z^+, /)$ a lattice?

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Without using the truth table, find the PCNF and PDNF of the statement $(\urcorner \mathbf{P} \longrightarrow \mathbf{R}) \wedge (\mathbf{Q} \leftrightarrow \mathbf{P})$. (8)

   (ii) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (8)

Or

   (b) (i) Check whether the following set of premises are not valid: Whenever the system software is being upgraded, users cannot access the file system. If users can access the file systems, then they can save new files. If users cannot save new files, then the system software is not being upgraded. (8)

(ii) Use the indirect method to prove that the conclusion $(\exists z)Q(z)$ follows from the premises $(\forall x)(P(x) \rightarrow Q(x))$ and $(\exists y)P(y)$. (8)

17. (a) (i) Prove, by mathematical induction, that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$. (8)

(ii) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$, given that $a_0 = 2$ and $a_1 = 8$. (8)

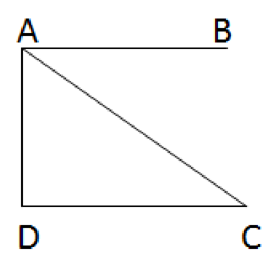
Or

(b) (i) Show that by mathematical induction principle, $3^{2n} + 4^{n+1}$ is divisible by 5, for $n \geq 0$. (8)

(ii) Find the number of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 and 7. (8)

18. (a) (i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (8)

(ii) Define Incidence matrix and path matrix. Also for the graph given below find all the possible paths of length 4 from vertex B to D. (8)



Or

(b) (i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (8)

(ii) Find the adjacency matrix of the following graph G. Find A^2, A^3 and $Y = A + A^2 + A^3 + A^4$. What is your observation of entries in A^2 and A^3 ? (8)

19. (a) (i) State and prove Lagrange's theorem. (8)

(ii) Define subgroup with an example. Also prove that the intersection of two subgroups of a group is also a subgroup of the group. (8)

Or

(b) (i) Let G be a group. If $a, b \in G$, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ (8)

(ii) Define subgroup with an example. Also prove that the intersection of two subgroups of a group is also a subgroup of the group. (8)

20. (a) (i) State and prove DeMargon's law of lattice. (8)

(ii) State and prove distributive inequality of Lattice. (8)

Or

(b) (i) In a complemented, distributive lattice, prove the following:

(i) $(ab)' + (a+b)' = a' + b'$ (ii) $ab'c + ab'c = b'c$ (8)
