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Question Paper Code: 44024

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Electronics and Communication Engineering

14UMA424 - PROBABILITY AND RANDOM PROCESS

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. If X is a Random Variable, where Var(x) =4, then predict the Var (3X+8)
   (a) 36 (b) 0 (c) 26 (d) 44

2. In which probability distribution, Variance and Mean are equal?
   (a) Binomial (b) Poisson (c) Geometric (d) None of these

3. When will the two Regression Lines be coincide
   (a) r= 0 (b) r= 1 (c) r= ± 1 (d) r= ∞

4. The conditional distribution of X given Y is
   (a) $f(x/y) = f(x, y) / f(x)$ (b) $f(x/y) = f(x, y) / f(y)$
   (c) $f(x/y) = f(x, y) f(x)$ (d) $f(x/y) = f(x, y) f(y)$

5. Every Strongly stationary process of order 2 is a
   (a) Orthogonal process (b) Stationary Process
   (c) WSS Process (d) None of these

6. If both T and S are discrete, then the random process is called
   (a) stationary (b) discrete random sequence
   (c) random process (d) Poisson process

7. $R_{XX}(\tau)$ is an _________ function of τ
   (a) positive (b) 1 (c) even (d) odd

8. The power spectral density of *X (t)* is defined by

(a) $y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$

(b) $X(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$

(c) $s_{XX}(\omega)=\int_{-\infty}^{\infty} R_{XX}(\tau)e^{i\omega\tau}d\tau$

(d) $s_{XX}(\omega)=\int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}d\tau$

9. Which of the following system is Causal?

(a) *y(t)=x(t+a)*

(b) *y(t)= x(t-a)*

(c) *(t)= a x(t+a)*

(d) *y(t)= x(t) – x(t-a)*

10. White noise is also called as

(a) system

(b) white Gaussian noise

(c) functional white noise

(d) stationary

PART - B (5 x 2 = 10 Marks)

11. State Axioms of Probability.

12. Define covariance.

13. Define random telegraph signal process.

14. State Winear–Khinchine theorem.

15. If *X(t)* is a WSS process and if $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$, then formulate $R_{XY}(\tau) = R_{XY}(\tau) * h(\tau)$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) In a bolt factory machines *A*, *B*, *C* manufacture respectively 25%, 35% and 40% of the total. Of their total output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines *A*, *B* and *C*? (8)

(ii) A random variable *X* has the p.d.f given by $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>0 \end{cases}$.

Find the MGF and mean. (8)

Or

(b) A random variable X has the following probability function

Value of x	0	1	2	3	4
$P(x)$	k	$3k$	$5k$	$7k$	$9k$

Find the value of k, $P(x < 3)$ and distribution function of x. (16)

17. (a) (i) The joint probability density function of a bivariate random variable (X, Y) is $f(x,y)=\begin{cases} k(x+y), 0<x<2, 0<y<2 \\ 0 \quad , elsewhere \end{cases}$. Find (1) the value of k (2) the marginal probability density of x and y (3) x and y independent. (8)

(ii) The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. The variance of x is 9. Evaluate the mean values of x and y and the Correlation coefficient between x and y. (8)

Or

(b) (i) The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. The variance of$x$ is 9. Evaluate the mean values of x and y and the Correlation coefficient between x and y. (8)

(ii) If X and Y are independent variants uniformly distributed in (0, 1). Identify the distribution of XY. (8)

18. (a) (i) Enumerate that the process $X(t)=A\cos\omega t+B\sin\omega t$ is Wide –Sense Stationary ,where A and B are random variables if $E(A)=E(B)=0$ and $E(A^2)=E(B^2)$ and $E(AB)=0$ (8)

The transition probability matrix of a Markov chain $\{X_n\}$,n = 1,2,3,…. having three states1, 2 and 3 is $P=\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)}= (0.7, 0.2, 0.1)$.

Identify i) $P[X_2=3]$, ii) $P[X_3=2,X_2=3,X_1=3,X_0=2]$ (8)

Or

(b) Generalize the postulates of a Poisson process and derive the probability distribution for the Poisson Process. Also Show that the sum of two independent Poisson process is again a Poisson process. (16)

19 (a) If the power spectral density of a WSS process is given by

$$S(\omega)=\begin{cases}\frac{b}{a}(a-|\omega|), & for\ |\omega|\le a\\ 0, & for\ |\omega|>a\end{cases}$$

Find the autocorrelation function of the process. (16)

Or

(b) (i) State and Prove Wiener-Khinchine theorem. (8)

(ii) Given that the autocorrelation function of a stationary random process is $R_{xx}(\tau)=\frac{25\tau^2+36}{6.25\tau^2+4}$. Predict the mean and variance of the process {X(t)}. (8)

20. (a) If *{X(t)}* is a WSS process and if $y(t)=\int_{-\infty}^{\infty}h(u)X(t-u)du$ then Prove that

(i) $R_{XY}(\tau)=R_{XX}(\tau)*h(-\tau)$

(ii) $R_{YY}(\tau)=R_{XY}(\tau)*h(\tau)$

(iii) $S_{XY}(\omega)=S_{XX}(\omega)H^*(\omega)$

(iv) $S_{YY}(\omega)=S_{XX}(\omega)|H(\omega)|^2$ (16)

Or

(b) (i) A system has an impulse response h(t)=$e^{-\beta t}\ U(t)$, predict the power spectral density of the output Y(t) corresponding to the input X(t). (8)

(ii) If *X(t)* is a band limited process such that $S_{xx}(\omega)=0, |\omega|>\sigma$, then formulate $2[R_{xx}(0)-R_{xx}(\tau)]\le\sigma^2\tau^2R_{xx}(0)$. (8)