Reg. No. : | | | | | | | | | | |

Question Paper Code: 44021

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Computer Science and Engineering

14UMA421 - APPLIED STATISTICS AND QUEUEING NETWORKS

(Common to Information Technology)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. If x is a continuous random variable, then $\int_{-\infty}^{\infty} f(x)dx$ equals

   (a) 0 (b) 1 (c) -1 (d) 2

2. Given $E\left[X^2\right] = 6$, assuming x as a Poisson variate, the value of $E\left[X\right]$ is

   (a) 1 (b) 2 (c) 3 (d) 4

3. C_{XY} is the covariance of X and Y, then C_{XY} =

   (a) $E\left[XY\right] - E\left[X\right]E\left[Y\right]$ (b) $E\left[XY\right] + E\left[X\right]E\left[Y\right]$

   (c) $E\left[XY\right] * E\left[X\right]E\left[Y\right]$ (d) $E\left[XY\right] / E\left[X\right]E\left[Y\right]$

4. If $X_1, X_2, \ldots, X_n, \ldots$ is a sequence of independent RVs with $E(X_i) = \mu_i$ and $Var(X_i) = \sigma_i^2, i = 1,2, \ldots$ and if $S_n = X_1 + X_2 + \cdots + X_n$, then under certain general conditions S_n follows a

   (a) Binomial distribution (b) Poisson

   (c) Normal (d) Exponential

5. Latin square design are most widely used in the field of

(a) industry (b) medicine (c) agriculture (d) astronomy

6. In a 4 X 4 latin square, the total of such possibilities are

(a) 8 (b) 10 (c) 200 (d) 576

7. Average time a customer waits before being served

(a) W_s (b) W_q (c) L_s (d) L_q

8. Average time a customer waits before being served

(a) W_s (b) W_q (c) L_s (d) L_q

9. In Pollaczek – Khinchine formula, ρ =

(a) $\lambda E(T)$ (b) $\lambda^2 E(T)$ (c) $\lambda E^2(T)$ (d) $\lambda^2 E^2(T)$

10. No customer may enter the system from outside

(a) Jackson (b) Open Jackson

(c) Closed Jackson (d) None of these

PART - B (5 x 2 = 10 Marks)

11. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test in fewer than 4 trials?

12. The random variable (X, Y) have the joint p.d.f $f(x,y) = x + y \quad 0 \le x \le 1,\ 0 \le y \le 1$.Find the marginal density function of Y.

13. What do you understand by design of experiments?

14. A system has a single server which can accommodate maximum of 10 persons. If the arrival is 10 / hr and service is 5 minutes / per customer find the traffic intensity.

15. Define Open Jackson Networks?

PART - C (5 x 16 = 80 Marks)

16. (a) (i) The probability density function of a random variable X is given by

$$f(x) = \begin{cases} kx(2-x)^2 & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

Find K, Mean and variance of the distributions. (8)

(ii) Derive the M.G.F, Mean and variance of Gamma distribution. (8)

Or

(b) (i) A coin is tossed until a head appears or until it has been tossed three times. Given that the head does not appear on the first toss, find the probability that the coin is tossed three times. (8)

(ii) Suppose the height of men of a certain country are normally distributed with average 68 and standard deviation 2.5, find the percentage of country men who are (i) between 66 and 71 (ii) Approximately 6 fit tall. (8)

17. (a) (i) Given $f_{xy}(x,y) = cx\,(x-y), 0 < x < 2, -x < y < x, and \quad 0$ elsewhere, (1) evaluate 'c' (2) find $f_x(x)$ (3) $f_{y/x}(y/x)$ and (4) $f_y(y)$. (8)

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of U = X – Y. (8)

Or

(b) (i) The joint probability function (X,Y) is given by $P(x,y) = K(2x+3y), x = 0,1,2;\ \ y = 1,2,3.$ Find the marginal distributions and P(X + Y). (8)

(ii) Following table gives the data on rainfall and discharge in a certain river. Obtain the Line of regression of Y on X. (8)

Rainfall (X)	1.53	1.78	2.60	2.95	3.42
Discharge (Y)	33.5	36.3	40	45.8	53.5

18. (b) (i) The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machine.

		Machine Type			
		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

Find SSC, SSR, SSE for the above data. (12)

(ii) State difference between LSD and RBD. (4)

Or

(b) Analyze the variance in the Latin square of yields (in kgs) of paddy where P,Q, R, S denote the different methods of cultivation.

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields. ($F_{0.05}\,(3,6) = 4.76$). (16)

19. (a) (i) Explain Markovian Birth – Death process and obtain the expressions for steady state probabilities. (8)

(ii) A supermarket has two girls attending sales at the counters. If the service time for each customer is exponential with mean 4 min and if people arrive in Poisson fashion at the rate of 10 per hour. What is the probability that the customers has to wait for service? (8)

Or

(b) (i) In a railway marshalling yard, goods trains arrive at the rate of 30 trains per day. Assume that the inter-arrival time follows as exponential distribution and the service time is also to be assumed as exponential with mean of 36 minutes. Calculate (1) the probability that the yard is empty (2) the average queue length, assuming the line capacity of the year is 9 trains. (8)

(ii) A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Find the average number of cars in the service station, and the average number of cars waiting for service. (8)

20. (a) Derive Pollaczek – Khinchine Formula. (16)

Or

(b) (i) A car wash facility operates with only one bay. Cars arrive according to a poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. Find the average number of cars waiting in the parking lot, if the time for washing and cleaning a car follows (i) uniform distribution between 6 and 12minutes. (ii) a normal distribution with mean 12 minutes and S.D 3 minutes. (16)