Reg. No. : | | | | | | | | | | |

Question Paper Code: 43021

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The Fourier series expansion of an even function contains

   (a) Sine terms only (b) cosine terms only
   (c) Both sine and cosine terms (d) Neither cosine nor sine terms

2. The complex form of Fourier series of *f(x)* in $(-\ell, \ell)$ is given by, *f(x)* = $\sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}}$, where C_n is,

   (a) $\frac{2}{l}\int_{-l}^{l} f(x) e^{-\frac{in\pi x}{l}} dx$ (b) $\frac{1}{2l}\int_{-l}^{l} f(x) e^{-\frac{in\pi x}{l}} dx$

   (c) $\frac{2}{l}\int_{0}^{l} f(x) e^{-\frac{in\pi x}{l}} dx$ (d) $\frac{1}{2l}\int_{0}^{l} f(x) e^{\frac{in\pi x}{l}} dx$

3. $F\left(e^{-|x|}\right)$

   (a) does not exist (b) $\frac{2is}{(s^2+1)}$ (c) $\frac{1}{\sqrt{2\pi}}\frac{s}{(s^2+1)}$ (d) $\frac{1}{\sqrt{2\pi}}\frac{2}{(s^2+1)}$

4. Fourier sine transform of *xf(x)* is,

   (a) $F_c'(s)$ (b) $F_s'(s)$ (c) - $F_c'(s)$ (d) - $F_s'(s)$

5. $\lim_{z \to 1} (z-1) F(z) =$

(a) f(1) (b) $F(\infty)$ (c) $f(\infty)$ (d) f(0)

6. $Z\{cosn\theta\}$ is ________

(a) $\frac{(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (b) $\frac{Z\,(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (c) $\frac{Z}{Z^2-2Z\,cos\theta+1}$ (d) $\frac{1}{Z^2-2Z\,cos\theta+1}$

7. When the ends of a rod is non zero for one dimensional heat flow equation, the temperature function *u(x,t)* is modified as the sum of steady state and transient state temperatures. The transient part of the solution which,

(a) increases with increase of time (b) decreases with increase of time
(c) increases with decrease of time (d) increases with decrease of time

8. The two dimensional heat flow equation in steady state is,

(a) $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial u}{\partial t} = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ (d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

9. In solving equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, by Crank – Nicholson method, we take , $\frac{(\Delta x)^2}{\alpha^2 k}$, as

(a) ½ (b) 2 (c) 1 (d) 0

10. The standard five point formula in solving Laplace equation over a region is,

(a) $u_{ij} = \frac{1}{4}\left(u_{i-1,j+1} + u_{i-1,j} + u_{i+1,j} + u_{i+1,j-1}\right)$ (b) $u_{ij} = \frac{1}{4}\left(u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right)$

(c) $u_{ij} = \frac{1}{4}\left(u_{i-1,j-1} + u_{i+1,j+1} + u_{i+1,j-1} + u_{i-1,j+1}\right)$ (d) $u_{ij} = \frac{1}{4}\left(u_{i-1,j} + u_{i-1,j-1} + u_{i+1,j} + u_{i+1,j-1}\right)$

PART - B (5 x 2 = 10 Marks)

11. If *f(x)* = $|x|$, expanded as a Fourier series in $(-\pi, \pi)$, find a_0.

12. If *F(f(x)) = F(s)* , then prove that $F\left[\frac{d^n\,(f(x))}{dx^n}\right] = (-is)^n F(s)$

13. State initial and final value theorems of Z transforms.

14. Evaluate the steady state temperature of a rod of length ℓ whose ends are kept at 30^o and $40^o\ c$.

15. Derive the explicit difference equation corresponding to the partial differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Obtain the Fourier expansion of *f(x)*, given that $f(x) = \begin{cases} \sin x \ in & 0 \le x \le \pi \\ 0 \quad in & \pi \le x \le 2\pi \end{cases}$ and hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots$ (8)

(ii) Expand $f(x) = (x-1)^2$, $0 < x < 1$ in a Fourier series of sines only. (8)

Or

(b) (i) Find the cosine series for f (x) = x in (0, π) and then using Parseval's theorem, show that $\frac{1}{1^4} + \frac{1}{3^4} + \ldots. = \frac{\pi^4}{96}$. (8)

(ii) Find the complex form of Fourier series of $f(x)$ if $f(x) = \sin ax \ in -\pi < x < \pi$. (8)

17. (a) (i) Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$. (8)

(ii) Find Fourier cosine transform of $e^{-a^2x^2}$. (8)

Or

(b) (i) Show that Fourier Transform of $f(x) = e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(4+x^2)\ (25+x^2)}$ using Fourier transform method. (8)

18. (a) (i) Find Z $(t^2 e^{-t})$ and Z $(\sin^3 \frac{n\pi}{6})$ (8)

(ii) Find Z $^{-1}\left(\frac{z^2 - 3z}{(z-5)(z+2)}\right)$ using residue theorem. (8)

Or

(b) (i) Solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$ and $y_1 = 1$. (8)

(ii) Find the inverse Z – transform of $\frac{z(z+1)}{(z-1)^3}$ by residue method. (8)

19. (a) A string is stretched between two fixed points at a distance 2ℓ apart and the points of the string are given initial velocities v where,

$$V = \begin{cases} \frac{cx}{l} & in\ 0 < x < l \\ \frac{c(2l - x)}{l} & in\ l < x < 2l \end{cases}$$

, x being the distance from an end point. Find the displacement of the string at any subsequent time. (16)

Or

(b) A metal bar 20cm long, with insulated sides, has its ends A and B kept at 30ºC and 90ºC respectively until steady state conditions prevail. The temperature at each end is suddenly raised to 0ºC and kept so. Find the subsequent temperature at any time of the bar at any time. (16)

20. (a) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units; satisfying the following boundary conditions:

(i) $u(0, y) = 0$ for $0 \leq y \leq 4$

(ii) $u(4, y) = 12 + y$ for $0 \leq y \leq 4$

(iii) $u(x, 0) = 3x$ for $0 \leq x \leq 4$

(iv) $u(x, 4) = x^2$ for $0 \leq x \leq 4$ (16)

Or

(b) (i) Solve $u_{xx} = 32\ u_t$ with h=0.25 for t>0; 0<x<1 and u(x,0)=u(0,t)=0; u(1,t)=t .Tabulate u upto t=5 sec using Bender-Schmidt formula. (8)

(ii) Find the solution to the wave equations $u_{xx} = u_{tt}$,0 < x < 1, t > 0, given that $u_t(x,0)=0$, $u(1,t) = u(0,t) = 0$ and $u(x,0) = 100 \sin \pi x$. Compute u for 4 time steps with h=0.25. (8)
