Reg. No. : | | | | | | | | | | |

Question Paper Code: 42002

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Civil Engineering

14UMA202 - ENGINEERING MATHEMATICS – II

(Common to ALL Branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. The complete solution of $(D^3 - D)y = 0$ is
   (a) $y = A + Bx + Cx^2$ (b) $y = A + Bcosx + Csinx$
   (c) $y = A + Be^x + Ce^{-x}$ (d) $y = Ax + Be^{-x} + Ce^x$

2. The complimentary function of $(D^2 - 2D)y = 3e^x \sin x$ is
   (a) $(A + Bx)e^{2x}$ (b) $(Ax + B)e^{-2x}$ (c) $A + Be^{2x}$ (d) $Ae^x + Be^{2x}$

3. If $\vec{F} = (4x+y)\vec{i} + 2y\vec{j} - 3az\vec{k}$ is Solenoidal, then a =
   (a) 0 (b) -1 (c) 1 (d) 2

4. By stokes theorem, $\int_c \vec{r}\,\overrightarrow{dr} =$ _______
   (a) π (b) 1 (c) 0 (d) None of these

5. The fixed points of $\omega = \dfrac{3z-4}{z-1}$ is
   (a) 2, -2 (b) 2, 0 (c) 0, 2 (d) 2, 2

6. The bilinear transformation that maps the points $\infty, i, 0$ onto $0, i, \infty$ is

(a) $-\frac{1}{z}$ (b) $-\frac{i}{z}$ (c) $\frac{i}{z}$ (d) None of these

7. Which of the following is not an analytic function?

(a) $\sin z$ (b) z (c) $\sinh z$ (d) $\bar{z}$

8. Conformal mapping is a mapping which preserves angle

(a) in magnitude (b) in sense

(c) both in magnitude and sense (d) Either in magnitude or in sense

9. $L\left[\frac{1}{\sqrt{t}}\right] =$

(a) $\sqrt{\frac{\pi}{s}}$ (b) $\frac{\sqrt{\pi}}{s}$ (c) $-\sqrt{\frac{\pi}{s}}$ (d) $\frac{\pi}{\sqrt{s}}$

10. $L\left[\frac{\cos at}{t}\right] =$

(a) $\frac{1}{s}$ (b) 0 (c) $\frac{-1}{s^2}$ (d) None of these

PART - B (5 x 2 = 10 Marks)

11. Solve $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

12. Find the directional derivative of $\Phi = x^2yz + 4xz^2$ at the point (1,-2,-1) in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$.

13. Test whether the function e^{4z} is analytic or not.

14. State Cauchy's integral formula.

15. Find the Laplace transform of $\sin 3t \sin 5t$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Solve $(D^4 - 2D^2 + 1)y = (x + 1)e^{2x}$. (8)

(ii) Solve $\left(D^2 + 4\right)y = 4\tan 2x$ by the method of variation of parameters. (8)

Or

(b) (i) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$. (8)

(ii) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in 1 hour. What was the value of N after 3/2 hours? (8)

17. (a) (i) Prove $\vec{F} = (y^2 \cos x + z^3)\,\vec{i} + (2y\sin x - 4)\,\vec{j} + 3xz^2\,\vec{k}$ is irrotational and find its scalar potential. (8)

(ii) Verify Green's theorem in the plane for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region defined by $x = y^2, y = x^2$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = (x^2-yz)\vec{i} + (y^2-xz)\vec{j} + (z^2-xy)\vec{k}$ and S is the surface of the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b,$ $z = 0$ and $z = c$. (16)

18. (a) (i) If $w = u(x, y) + iv(x, y)$ is an analytic function the curves of the family $u(x, y) = a$ and the curves of the family $v(x, y) = b$ are cut orthogonally, where a and b are the constants. (8)

(ii) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$. (8)

Or

(b) (i) Show that the function $u = \log\sqrt{x^2 + y^2}$ is harmonic and also find its conjugate. (8)

(ii) Obtain the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 0, 1, \infty$ respectively. (8)

19 (a) Using contour integration, prove that $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta = \frac{\pi}{12}$. (16)

Or

(b) (i) Show that the function $u = log\sqrt{x^2 + y^2}$ is harmonic and also find its conjugate (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using contour integration. (8)

20. (a) Given $y' = x^2 + y$, $y(0) = 1$, find y (0.1) by Taylor series method, y (0.2) by modified Euler's method, y (0.3) by R-K method. (16)

Or

(b) (i) Find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$ using Convolution theorem. (8)

(ii) Using Runge-Kutta method of 4th order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0)=1$ at $x=0.2$. (8)