Reg. No. : | | | | | | | | | | |

Question Paper Code: 41002

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If the Eigen values of the matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$ are 2, -2 then the Eigen values of A^T are

   (a) $\frac{1}{2}, \frac{-1}{2}$ (b) 2, -2 (c) 1, -1 (d) 1, 3

2. If 0, 3, 4 are eigen values of a square matrix A of order 3 then $|A|$ =

   (a) 12 (b) 0 (c) ∞ (d) $\frac{1}{12}$

3. Examine the nature of the series $1 + 2 + 3 + 4 + \dots\dots\dots + n + \dots\dots \infty$

   (a) divergent (b) convergent (c) oscillatory (d) linear

4. D'Alembert's test is also called

   (a) Ratio test (b) Root test (c) Abel's test (d) none of these

5. The radius of curvature of the curve $y = e^x$ at (0,1) is

(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{3}$

6. The envelope of the family of lines $y = x + a/m$, m being a positive integer

(a) $y^2 = 4ax$ (b) $x^2 = 4ay$ (c) $x^2 + y^2 = a^2$ (d) $xy = a^2$

7. Let u and v be functions of x, y and $u = e^v$ .Then u and v are

(a) Functionally dependent (b) Functionally independent
(c) Functionally linear (d) Functionally non-linear

8. A stationary point of $f(x, y)$ at which $f(x, y)$ has neither a maximum nor a minimum is called

(a) Extreme point (b) Max-Min point
(c) Saddle point (d) Nothing can be said

9. The value of $\int_0^a \int_0^{\sqrt{a^2-x^2}} dy\, dx$ is

(a) $\frac{\pi a^2}{4}$ (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi a^2}{8}$ (d) πa^2

10. The value of the double integral $\int_0^{\pi} \int_0^{a} r\, dr\, d\theta$ is

(a) πa^2 (b) $\frac{\pi a^2}{2}$ (c) $\frac{\pi r^2}{2}$ (d) πr^2

PART - B (5 x 2 = 10 Marks)

11. Write down the quadratic form corresponding to the matrix $A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$

12. Test the convergence of the series $\sum_{1}^{\infty} \frac{n!2^n}{n^n}$ by D'Alembert's Ratio test.

13. Find the radius of curvature of the curve $y = e^x$ at $x = 0$.

14. If $x = u^2 - v^2$ and $y = 2uv$, find the Jacobian of x and y with respect to u and v.

15. Indicate the region of integration of $\int_0^a \int_{\frac{x^2}{a}}^{x} xy\,dy\,dx$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Using Cayley-Hamilton theorem, find A^4 for $A=\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ (8)

(ii) Find the eigen values and eigen vectors of $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (8)

Or

(b) Reduce the Q.F $x^2+y^2+z^2-2xy-2yz-2zx$ in to a canonical form by an orthogonal transformation. (16)

17. (a) (i) Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.6}+...$ (8)

(ii) Examine the convergence of the series $\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-...$ $(x>0)$ (8)

Or

(b) Prove that if $b-1>a>0$, the series $1+\frac{a}{b}+\frac{a(a+1)}{b(b+1)}+\frac{a(a+1)(a+2)}{b(b+1)(b+2)}+.....$ converges. (16)

18. (a) Prove that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta); y=a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$. (16)

Or

(b) Considering the evolute as the envelope of normals, find the evolute of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (16)

19. (a) If u is function of x and y; by changing to polar form with $x = r\cos\theta, y = r\sin\theta$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}$. (16)

Or

(b) (i) If $g(x, y) = \psi(u, v)$, where $u = x^2 - y^2$, $v = 2xy$, then prove that

$$\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=4(x^2+y^2)\left(\frac{\partial^2 \psi}{\partial u^2}+\frac{\partial^2 \psi}{\partial v^2}\right).$$ (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 where $y_1=\frac{x_2x_3}{x_1}$, $y_2=\frac{x_1x_3}{x_2}$, $y_3=\frac{x_1x_2}{x_3}$. (8)

20. (a) (i) By changing in to polar coordinates, evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy$. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, $x = 0$, $y = 0$ and $z = 0$. (8)

Or

(b) (i) Evaluate $\iint_S z^3 dS$, where is S is the positive octant of the surface of the sphere. (8)

(ii) Evaluate $\iiint_V xyz\,dx\,dy\,dz$, where V is the volume of space inside the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (8)
