Reg. No. : | | | | | | | | | | |

Question Paper Code: 45021

B.E/B.Tech. DEGREE EXAMINATION, APRIL 2019

Fifth Semester

Computer Science and Engineering

01UMA521 – DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Prove that $P \rightarrow Q \Leftrightarrow 7P \vee Q$

2. Define universal and existential quantifiers.

3. State Pigeonhole principle and its generalization.

4. In how many ways can integers 1 through 9 be permuted such that no odd integer will be in its natural position?

5. Define a complete graph.

6. Define spanning tree.

7. Define a field in an algebraic system.

8. Explain the Kernal of the homomorphism g.

9. Le $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $< X, \leq >$.

10. State the Isotonic property of a Lattice.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Show that $Q.V\ (P \wedge 7Q) \vee (7P \wedge 7Q)$ is a tautology. (8)

(ii) Obtain PDNF of $(P \wedge Q)\ V\ (7P \wedge R)\ V\ (Q \wedge R)$. Also find PCNF. (8)

Or

(b) Show that RVS follows logically from the premises CVD, $CVD \rightarrow 7H$, $7H \rightarrow A \wedge 7B$ and $(A \wedge 7B) \rightarrow (RVS)$. (16)

12. (a) (i) How many bit strings of length 10 contain

(1) exactly four 1's (2) at most four 1's (3) at least four 1's

(4) an equal number of 0's and 1's. (8)

(ii) A man hiked for 10 hours and covered a total distance of 45 *km*. It is known that he hilled 6 *km* in the first hour and only 3 *km* in the last hour. Show that he must have hiked at least 9 *km* within a certain period of 2 consecutive hours. (8)

Or

(b) (i) Solve the recurrence relation $a_n = 2a_{n-1} + 2^n$, $a_0 = 2$. (8)

(ii) Prove the principle of inclusion – exclusion using mathematical induction. (8)

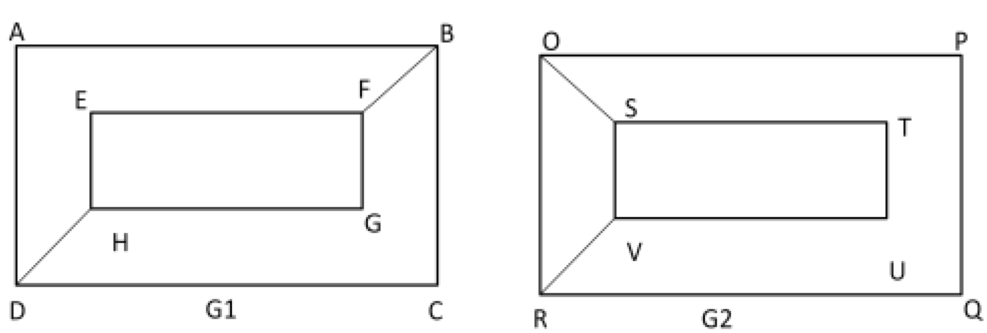
13. (a) (i) Describe a discrete structure based on a graph that can be used to model airline routes and their flight times. (8)

(ii) Define an Euler path and show that if a graph G has more than two vertices of odd degree, then there can be a no Euler path in G. (8)

Or

(b) (i) If all the vertices of an undirected graph are each of odd degree k, show that the number of edges of the graph is a multiple of K. (8)

(ii) Determine whether the graphs are isomorphic or not. (8)



14. (a) State and prove Lagrange's theorem. (16)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) If * is the binary operation on the set of real numbers defined by $a*b = a+b+2ab$, show that $(R,*)$ is a commutative monoid. (8)

15. (a) Show that the De Morgan's laws hold in a Boolean algebra . That is, show that for all x and y ,$\overline{(x \vee y)} = \bar{x} \wedge \bar{y}$ and $\overline{(x \wedge y)} = \bar{x} \vee \bar{y}$. (16)

Or

(b) (i) Let $(L,*,\oplus)$ be a distributive lattice. For any $a, b, c \, \varepsilon \, L$. prove that $(a*b = a*c) \wedge (a \oplus b = a \oplus c) => b = c$. (8)

(ii) In any Boolean algebra, show that $a = b$ if $a\bar{b}+\bar{a}b = 0$. (8)