Reg. No. : | | | | | | | | | | |

Question Paper Code: 34024

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

(Statistical tables may be permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. If a random variable X has the moment generating function $M(t) = \frac{3}{3-t}$, obtain the standard deviation of the variable X.

2. Two dice are thrown 720 times; find the average number of times in which the number on the first dice exceeds the number on the second dice.

3. State the equations of the two regression lines. What is the angle between them?

4. If $Y = -2X + 3$, find the $Cov(X, Y)$.

5. Prove that a first order stationary random process has a constant mean.

6. Consider a Markov chain with state {0, 1} transition probability matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Is the state 0 periodic? If so, what is the period?

7. State any two properties of cross correlation function.

8. State any two properties of an auto correlation function.

9. Define white noise.

10. Define average power in the response of a linear system.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) The contents of bags I, II, III with balls are as follows. 1 white, 2 black and 3 red; 2white, 1 black and 1 red; 4 white, 5 black and 3 red. One bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from bags I, II and III? (8)

(ii) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution. (8)

Or

(b) (i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act independently.

(1) What is the probability that all four lines are busy?

(2) What is the probability that atleast two of them are busy? (8)

(ii) Describe Gamma distribution, Obtain its moment generating function. Hence compute its mean and variance. (8)

12. (a) (i) The joint probability mass function of $p(X, Y) = k(2x + 3y)$, x = 0, 1, 2; y = 1, 2, 3. Find k and all the marginal and conditional probability distributions. Also find the probability distribution of $(X+Y)$. (8)

(ii) The joint probability density function of the two dimensional random variable (X, Y) is $(x,y) = \begin{cases} 2-x-y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & , else \text{ where} \end{cases}$. Find the correlation coefficient between X and Y. (8)

Or

(b) (i) The joint pdf of X and Y is given by $f(x,y) = e^{-(x+y)}, x > 0, y > 0$. Find the probability density function of $U = \frac{X+Y}{2}$. (16)

13. (a) Prove that the random process $[X(t)]$ with constant mean is mean ergodic, if

$$\lim_{T\to\infty} \int_{-T}^{T}\int_{-T}^{T} \frac{C(t_1, t_2)}{4T^2}\, dt_1 dt_2 = 0.$$ (16)

Or

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(b) (i) Define random telegraph signal process. Prove

(a) $P[X(t) = 1] = 1/2 = P[X(t) = -1]$, for all $t > 0$

(b) $E[X(t)] = 0$ and $Var[X(t)] = 1$ (8)

(ii) A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find

(1) the probability that he takes a train on the third day

(2) the probability that he drives to work in the long run. (8)

14. (a) State and prove Wiener-Khinchine theorem. (16)

Or

(b) (i) If $\{X(t)\}$ is a WSS process with auto correlation function $R_{XX}(\tau)$ and if $Y(t) = X(t+a) - X(t-a)$, show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$. (8)

(ii) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$. (8)

15. (a) (i) Prove that if the input to a time-invariant , stable linear system is a WSS process, then the output will also be a WSS process. (8)

(ii) If the input $x(t)$ and the output $y(t)$ are connected by the differential equation $T\frac{dy(t)}{dt} + y(t) = x(t)$, prove that they can be related by means of a convolution type integral, assuming that $x(t)$ and $y(t)$ are zero for $t \leq 0$. (8)

Or

(b) (i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ *and* $S_{YY}(\omega)$ are the power spectral density functions of the input *X(t)* and the output *Y(t)* and $H(\omega)$ is the system transfer function. (8)

(ii) The input to the RC filter is a white noise process with ACF $R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$. If the frequency response $H(\omega) = \frac{1}{1 + j\omega RC}$, find the autocorrelation and the mean-square value of the output process Y(t). (8)