Reg. No. : | | | | | | | | | | |

Question Paper Code: 33021

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. State the conditions for $f(x)$ to have Fourier series expansion.

2. If $f(x)=\sin x, \quad in -\pi < x < \pi$, find the values of a_0 and a_0?

3. Define Fourier Integral theorem.

4. Find the Fourier sine transform of $\cos x, 0 < x < a$.

5. Find the z - transform of $\{n\}$.

6. Write the formula for $Z^{-1}[F(z)]$ using Cauchy's residue theorem.

7. State initial and final value theorems on z - transform.

8. Write the appropriate solution of the one dimensional heat flow equation.

9. State the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.

10. Write a difference formula for solving one dimensional wave equation $u_{tt} = a^2 u_{xx}$.

PART - B (5 x 16 = 80 Marks)

11\. (a) Expand $f(x) = x^2$ when $-\pi \le x \le \pi$ in a Fourier series of periodicity 2π. Hence deduce that

(i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}\text{ .... } = \frac{\pi^2}{12}$

(ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{........} = \frac{\pi^2}{6}$. (16)

Or

(b) (i) Find the Half range cosine series for *y* = *x* in *(0, l)* and hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \text{......}\infty$. (8)

(ii) Compute the first two harmonics of the Fourier series of *f(x)* given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

12\. (a) (i) Find the Fourier transform of $f(x) = \begin{Bmatrix} 1-|x| \; : \; |x|<1 \\ 0 : otherwise \end{Bmatrix}$ and hence find the value of $\int_0^\infty \frac{sin^4 t}{t^4}\, dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x\, e^{-x^2}$. (8)

Or

(b) (i) Find the Fourier transform of $f(x) = \begin{Bmatrix} 1-|x| \; : \; |x|<1 \\ 0 : otherwise \end{Bmatrix}$ and hence find the value of $\int_0^\infty \frac{sin^4 t}{t^4}\, dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x\, e^{-x^2}$. (8)

13\. (a) (i) Find the Z transform of $a^n\, cosn\pi \;\; and \; e^t\, sin2t$. (8)

(ii) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = y_1 = 0$, using Z transforms. (8)

Or

(b) (i) State and prove initial and final value theorem on Z- transform. (8)

(ii) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by using method of Partial fraction. (8)

14. (a) The ends A and B of a rod l cm long have the temperature at 30^0c and 80^0c until steady state prevails. The temperature of the ends is then changed to 40^0c and 60^0c respectively. Find the temperature distribution in the rod at any time. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite length. If the temperature at short edge $y = 0$ is given by $u = \begin{cases} 20\,x & : 0 \le x \le 5 \\ 20(10-x) : & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at 0°C. Find the steady state temperature at any point of the plate. (16)

15. (a) Solve numerically $4u_{xx} = u_{tt}$ with the boundary conditions $u(0,t) = 0$, $u(4,t) = 0$ and the initial conditions $u_t(x,0) = 0$ and $u(x,0) = x\ (4-x)$ taking $h = 1$ up to *4* time steps. (16)

Or

(b) (i) Solve, by Crank-Nicholson method, the equation $u_{xx} = u_t$ subject to $u(x,0) = 0, u(0,t) = 0\ and\ u(1,t) = t$ for two time steps by taking $h = 0.25$. (8)

` (ii) Evaluate the pivotal values of the following equation taking $h = 1$ and up to one half of the period of the oscillation $16u_{xx} = u_{tt}$ given $u(0,t) = 0$, $u(5,t) = 0$, $u(x,0) = x^2(5-x)\ and\ u_t(x,0) = 0$. (8)