Reg. No. : | | | | | | | | | | |

Question Paper Code: 32002

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Civil Engineering

01UMA202 - ENGINEERING MATHEMATICS - II

(Common to all branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Solve $(x^2 D^2 + xD - 1)\, y = 0$.

2. Find the $P.I$ of $(D^3 - 1)\, y = e^{2x}$.

3. State Green's theorem.

4. Prove that div $\vec{r} = 3$.

5. Check whether xy^2 is real part of an analytic function.

6. Find the fixed points of $w = \dfrac{3z-4}{z-1}$.

7. State Cauchy's integral formula for first derivative of an analytic function.

8. Expand $\dfrac{1}{z-2}$ at $z = 1$ in a Taylor's series.

9. Find $L[e^t \sin 2t]$.

10. Define existence conditions of Laplace transform.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Solve: $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$. (8)

(ii) Solve $(D^2+4)\,y = x \sin x$. (8)

Or

(b) (i) Solve $(D^2+2D+5)\,y = e^{-x} \tan x$ by method of variation of parameter. (8)

(ii) Solve $\frac{dx}{dt} + y = \sin t$ and $\frac{dy}{dt} + x = \cos t$ given x = 2, y = 0 when t = 0. (8)

12. (a) Verify divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where S is the surface formed by the planes x = 0, x = a, y = 0, y = b, z = 0 and z = c. (16)

Or

(b) Verify Stoke's theorem for $\overline{F} = y\vec{i} + z\vec{j} + x\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (16)

13. (a) (i) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and determine its harmonic conjugate. (8)

(ii) Prove that the analytic function with constant modulus is also constant. (8)

Or

(b) (i) If $f(z) = u + iv$ an analytic function and $u - v = e^x(\cos y - \sin y)$ find $f(z)$ interms of z. (8)

(ii) Find the image of $|z - 2i| = 2$, under the transformation $w = 1/z$. (8)

14. (a) (i) Evaluate $\int_c \frac{z+4}{z^2+2z+5}\,dz$, where c is the circle $|z+1-i| = 2$ using Cauchy's integral formula. (8)

(ii) Using residue theorem, evaluate $\int_c \frac{3z^2+z-1}{(z^2-1)(z-3)}\,dz$ where c is the circle $|z| = 2$. (8)

Or

(b) (i) Find Laurent's series expansion of f(z) = $\dfrac{7z-2}{z(z-2)(z+1)}$ valid 1< |z+1|<3 (8)

. (ii) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$. (8)

15. (a) (i) Find the Laplace transform of $\dfrac{e^{-t}\sin 2t}{t}$. (8)

(ii) Find the Laplace transform of the square wave function defined by

$$f(t)=\begin{cases} 1; & 0<t<a \\ -1; & a<t<2a \end{cases}$$ (8)

Or

(b) (i) Find the Laplace transform of the Half-wave rectifier function

$$f(t)=\begin{cases} \sin\omega t, & 0<t<\dfrac{\pi}{\omega} \\ 0, & \dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega} \end{cases}$$, with $f\left(t+\dfrac{2\pi}{\omega}\right)=f(t)$. (8)

(ii) Solve the initial value problem $y''-3y'+2y=4t,\ y(0)=1,\ y'(0)=-1.$ (8)