Reg. No. : | | | | | | | | | | |

Question Paper Code: 31002

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

First Semester

Civil Engineering

01UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2013)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Two of the Eigen values of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the Eigen value of A^{-1}.

2. State Cayley – Hamilton theorem and its uses.

3. Find the equation of the sphere with centre (2, 3, 5) and touches the XoY – plane.

4. Define the right circular cylinder.

5. Find the curvature of the curve $2x^2+2y^2+5x-2y+1=0$.

6. Find the envelope of the family of curve $y = mx + \frac{a}{m}$.

7. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

8. If $x = r\cos\theta$ and $y = r\sin\theta$, then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

9. Evalute $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r \, d\theta \, dr$.

10. Evaluate $\int_0^1 \int_0^2 \int_0^e dz\, dy\, dx$.

PART - B (5 x 16 = 80 Marks)

11. (a) Find the Eigen values and Eigenvectors of the matrix $A=\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (16)

Or

(b) Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ to canonical form by orthogonal reduction. Also find the nature of the quadratic form. (16)

12. (a) (i) Find the equations of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x+2y-z+5=0$. (8)

(ii) Find the centre and radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+17 = 0$. Also find the equation of the sphere having the above circle as great circle. (8)

Or

(b) (i) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (8)

(ii) Find the equation of the right circular cylinder whose axis is the line x = 2y = -z and radius 4. (8)

13. (a) (i) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. (8)

(ii) Find the evolute of the parabola $x^2=4ay$. (8)

Or

(b) (i) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters 'a' and 'b' are connected by the relation $a+b = c$. (8)

(ii) Find the envelope of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, considering it as the envelope of normals. (8)

14. (a) (i) Given the transforms $u = e^x \cos y$ & $v = e^x \sin y$ and that ϕ is a function of u & v and also of x & y, prove that $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = (u^2+v^2)\left(\frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}\right)$. (8)

(ii) Expand $e^x \cos y$ in powers of x and y as far as the terms of third degree using Taylor's expansion. (8)

Or

(b) (i) Examine $f(x,y) = x^3 + y^3 - 12x - 3y + 20$ for its extreme values. (8)

(ii) Find the dimensions of the rectangular box without a top of maximum capacity, whose surface is 108 *sq.cm.*. (8)

15. (a) (i) Change the order of the integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dxdy$. (8)

(ii) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)\,dydx$ by changing into polar coordinates. (8)

Or

(b) (i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx\, dy$ by changing into polar coordinates. (8)

(ii) Find the volume of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)