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Question Paper Code: 53021

B.E. / B.Tech. DEGREE EXAMINATION, APRIL 2019

Third Semester

Civil Engineering

15UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, EIE, MECH, Chemical, Biomedical and

Agriculture Engineering Branches)

(Regulation 2015)

Duration: Three hours

Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

- Fourier coefficients for odd function $f(x)$ defined in interval $-\pi \leq x \leq \pi$ and $f(x+2\pi)$ are CO1- R
 - $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx, a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx, b_n = 0$ (b) $a_0 = 0, a_n = 0, b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx$
 - $a_0 = 0, a_n = 0, b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$ (d) $a_0 = 0, a_n = 0, b_n = 0$
- The root mean square value of $f(x) = x$ in $(0, 1)$ interval CO1- R
 - $2/3$ (b) $1/(3)^{1/2}$ (c) $2/(3)^{1/2}$ (d) $4/5$
- Product of two even or two odd functions is CO2- R
 - even (b) odd (c) symmetric (d) antisymmetric
- Give a function which is self reciprocal under sine transform CO2- R
 - x (b) x^2 (c) $1/(x)^{(1/2)}$ (d) $1/(x)^{(3/2)}$
- Find $Z[a^{n-1}]$ CO3- R
 - $\frac{az}{z-1}$ (b) $\frac{1}{z-1}$ (c) $\frac{z^2}{z-a}$ (d) $1/a \left(\frac{z}{z-a} \right)$

6. $z(au_n + bv_n) = az(u_n) + bz(v_n)$ is CO3- R
- (a) Damping rule (b) Recurrence (c) Linear property (d) Shifting property
property
7. The p.d.e of $z = ax+by$ is CO4- R
- (a) $x+y$ (b) $qx+py$ (c) $px+qy$ (d) $x-y$
8. $r + t = x^2 + y$ is a partial differential equation of order CO4- R
- (a) 0 (b) 1 (c) 1,2 (d) 2
9. What is the constant a^2 in the wave equation CO5- R
- (a) $a^2 = \frac{T}{m}$ (b) $a^2 = \frac{1}{m}$ (c) $a^2 = \frac{T}{2}$ (d) $a^2 = \frac{T^2}{m}$
10. Let $u_{xx} + u_{yy} = 0$ is CO5- R
- (a) wave equation (b) heat equation (c) Laplace equation (d) Poisson equation

PART – B (5 x 2= 10 Marks)

11. Write the conditions for a function $f(x)$ to satisfy for the existence of a Fourier series CO1- R
12. Find the Fourier cosine transform of $e^{-ax}, x \geq 0$ CO2- R
13. Define Z transform of the sequence $\{f(n)\}$ CO3- R
14. Form the p.d.e by eliminating the arbitrary constant $z = ax + by$ CO4- R
15. Classify the pde $3u_{xx} + 4u_{xy} + 3u_y - 2u_x = 0$ CO5- R

PART – C (5 x 16= 80Marks)

16. (a) (i) Find the Fourier series expansion of the series CO1- App (8)

$$f(x) = \begin{cases} x & , 0 \leq x < \pi \\ 2\pi - x & , \pi < x \leq \pi \end{cases} \text{ with period } 2\pi. \text{ Deduce that}$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$$

- (ii) Obtain the Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$ with CO1- App (8)
period 2π . Hence deduce that

$$(i) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots = \frac{\pi^2}{12} \quad (ii) \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

$$(iii) \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Or

- (b) Determine the first two harmonic of the Fourier series for the following values. CO1- App (16)

X:	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
Y:	1.98	1.30	1.05	1.30	-0.88	-0.25

17. (a) Find the Fourier transform of CO2- App (16)

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

Hence deduce the following:

$$(i) \int_0^{\infty} \left(\frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}$$

$$(ii) \int_0^{\infty} \left(\frac{\sin t}{t} \right)^4 dt = \frac{\pi}{3}$$

Or

- (b) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ Hence CO2- App (16)

deduce that $\int_0^{\infty} \left(\frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}$.

18. (a) Find the Inverse Z-transform of $x(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}$ using CO3- Ana (16)
convolution theorem.

Or

- (b) Solve CO3- Ana (16)

$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$, using Z – transform.

19. (a) (i) Solve $(y+z)p+(z+x)q=x+y$ CO4-App (8)
- (ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = x^2y + \text{Sin}(x+2y)$ CO4-App (8)
- Or
- (b) Solve $z = px+qy + \sqrt{1+p^2+q^2}$ CO4- App (16)
20. (a) A string is stretched and fastened to two points $x = 0$ and $x = 1$ CO5- U (16)
 apart. Motion is started by displacing the string into the form $y = \mu x (1 - x)$ from which it is released at time $t = 0$. Find the displacement of the string at any time 't'
- Or
- (b) A rod of length 1 has its end A & B kept at 0°C and 10°C CO5- U (16)
 respectively until steady state condition prevails. If the temperature at B is reduced to 0°C and kept so, while that of A is maintained, find the temperature $u(x,t)$